PROBLEMS FOR NOVEMBER

Please send your solutions to

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no later than December 5, 2008. Electronic files can be sent to *barbeau@math.utoronto.ca*. However, if you respond electronically, please do not scan a handwritten solution. The attachment uses an inordinate amount of space and often causes difficulties in downloading. Also, the handwriting is frequently indistinct or splotchy. Please type the solution using a word-processing package (TeX is good and can be sent in a pdf file). It is important that your complete mailing address and your email address appear legibly on the front page. If you do not write your family name last, please underline it.

577. $ABCDEF$ is a regular hexagon of area 1. Determine the area of the region inside the hexagon thst belongs to none of the triangles ABC, BCD, CDE, EFA and FAB.

578. $ABEF$ is a parallelogram; C is a point on the side AE and D a point on the aide BF for which $CD \| AB$. The sements CF and EB intersect at P; the segments ED and AF intersect at Q. Prove that $PQ \| AB$.

579. Solve, for real x, y, z the equation

$$\frac{y^2 + z^2 - x^2}{2yz} + \frac{z^2 + x^2 - y^2}{2zx} + \frac{x^2 + y^2 - z^2}{2xy} = 1 \ .$$

580. Two numbers m and n are two perfect squares with four decimal digits. Each digit of m is obtained by increasing the corresponding digit of n be a fixed positive integer d. What are the possible values of the pair (m, n).

581. Let $n \geq 4$. The integers from 1 to n inclusive are arranged in some order around a circle. A pair (a, b) is called *acceptable* if $a < b$, a and b are not in adjacent positions around the circle and at least one of the arcs joining a and b contains only numbers that are less than both a and b. Prove that the number of acceptable pairs is equal to $n - 3$.

582. Suppose that f is a real-valued function defined on the closed unit interval $[0, 1]$ for which $f(0) = f(1) = 0$ and $|f(x) - f(y)| < |x - y|$ when $0 \leq x < y \leq 1$. Prove that $|f(x) - f(y)| < \frac{1}{2}$ for all $x, y \in [0, 1]$. Can the number $\frac{1}{2}$ in the inequality be replaced by a smaller number and still result in a true proposition?

583. Suppose that $ABCD$ is a convex quadrilateral, and that the respective midpoints of AB, BC, CD, DA are K, L, M, N. Let O be the intersection point of KM and KN. Thus $ABCD$ is partitioned into four quadrilaterals. Prove that the sum of the areas of two of these that do not have a common side is equal to the sum of the areas of the other two, to wit

$$[AKON] + [CMOL] = [BLOK] + [DNOM] \ .$$