PROBLEMS FOR DECEMBER

Please send your solutions to

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no later than January 5, 2008. Electronic files can be sent to *rosumihai@yahoo.ca*. However, please do not send scanned files; they use a lot of computer space, are often indistinct and can be difficult to download. It is important that your complete mailing address and your email address appear legibly on the front page. If you do not write your family name last, please underline it.

584. Let n be an integer exceeding 2 and suppose that $x_1, x_2, \cdots, x_n$ are real numbers for which $\sum_{i=1}^n x_i = 0$ and $\sum_{i=1}^n x_i^2 = n$. Prove that there are two numbers among the x_i whose product does not exceed -1.

585. Calculate the number
$$a = \lfloor \sqrt{n-1} + \sqrt{n} + \sqrt{n+1} \rfloor^2 \ ,$$
where $\lfloor x \rfloor$ denotes the largest integer than does not exceed x and n is a positive integer exceeding 1.

586. The function defined on the set $\mathbf{C*}$ of all nonzero complex numbers satisfies the equation
$$f(z)f(iz) = z^2 \ ,$$
for all $z \in \mathbf{C}^*$. Prove that the function $f(z)$ is odd, *i,e.*, $f(-z) = -f(z)$ for all $z \in \mathbf{C}^*$. Give an example of a function that satisfies this condition.

587. Solve the equation
$$\tan 2x \tan \left(2x + \frac{\pi}{3} \right) \tan \left(2x + \frac{2\pi}{3} \right) = \sqrt{3} \ .$$

588. Let the function $f(x)$ be defined for $0 \leq x \leq \pi/3$ by
$$f(x) = \sec \left(\frac{\pi}{6} - x \right) + \sec \left(\frac{\pi}{6} + x \right) \ .$$
Determine the set of values (its image or range) assumed by the function.

589. In a circle, A is a variable point and B and C are fixed points. The internal bisector of the angle BAC intersects the circle at D and the line BC at G; the external bisector of the angle BAC intersects the circle at E and the line BC at F. Find the locus of the intersection of the lines DF and EG.

590. Let $SABC$ be a regular tetrahedron. The points M, N, P belong to the edges SA, SB and SC respectively such that $MN = NP = PM$. Prove that the planes MNP and ABC are parallel.