PROBLEMS FOR MAY

Please send your solution to

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no later than June 30, 2007. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

493. Prove that there is a natural number with the following characteristics: (a) it is a multiple of 2007; (b) the first four digits in its decimal representation are 2009; (c) the last four digits in its decimal representation are 2009.

494. (a) Find all real numbers x that satisfy the equation

$$(8x - 56)\sqrt{3 - x} = 30x - x^2 - 97 \ .$$

(b) Find all real numbers x that satisfy the equation

$$\sqrt{x} + \sqrt[3]{x + 7} = \sqrt[4]{x + 80} \ .$$

495. Let $n \geq 3$. A regular $n-$gon has area S. Squares are constructed externally on its sides, and the vertices of adjacent squares that are not vertices of the polygon are connectd to form a $2n-$sides polygon, whose area is T. Prove that $T \leq 4(\sqrt{3} + 1)S$. For what values of n does equality hold?

496. Is the hundreds digit of $N = 2^{2006} + 2^{2007} + 2^{2008}$ even or odd? Justify your answer.

497. Given $n \geq 4$ points in the plane with no three collinear, construct all segments connecting two of these points. It is known that the length of each of these segments is a positive integer. Prove that the lengths of at least 1/6 of the segemtns are multiples of 3.

498. Let a be a real parameter. Consider the simultaneous sytem of two equations:

$$\frac{1}{x + y} + x = a - 1 \ ; \tag{1}$$

$$\frac{x}{x + y} = a - 2 \ . \tag{2}$$

(a) For what value of the parameter a does the system have exactly one solution?

(b) Let $2 < a < 3$. Suppose that (x, y) satisfies the sytem. For which value of a in the stated range does $(x/y) + (y/x)$ reach its maximum value?

499. The triangle ABC has all acute angles. The bisector of angle ACB intersects AB at L. Segments LM and LN with $M \in AC$ and $N \in BC$ are constructed, parpendicular to the sides AC and BC respectively. Suppose that AN and BM intersect at P. Prove that CP is perpendicular to AB.