PROBLEMS FOR NOVEMER

Please send your solution to

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no later than December 21, 2006. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

465. For what positive real numbers a is

$$\sqrt[3]{2+\sqrt{a}} + \sqrt[3]{2-\sqrt{a}}$$

an integer?

466. For a positive integer m, let $\overline{m}$ denote the sum of the digits of m. Find all pairs of positive integers $(m.n)$ with $m < n$ for which $(\overline{m})^2 = n$ and $(\overline{n})^2 = m$.

467. For which positive integers n does there exist a set of n distinct positive integers such that

(a) each member of the set divides the sum of all members of the set, *and*

(b) none of its proper subsets with two or more elements satisfies the condition in (a)?

468. Let a and b be positive real numbers satisfying $a + b \geq (a - b)^2$. Prove that

$$x^a(1-x)^b + x^b(1-x)^a \leq \frac{1}{2^{a+b-1}}$$

for $0 \leq x \leq 1$, with equality if and only if $x = \frac{1}{2}$.

469. Solve for t in terms of a, b in the equation

$$\sqrt{\frac{t^3+a^3}{t+a}} + \sqrt{\frac{t^3+b^3}{t+b}} = \sqrt{\frac{a^3-b^3}{a-b}}$$

where $0 < a < b$.

470. Let ABC, ACP and BCQ be nonoverlapping triangles in the plane with angles CAP and CBQ right. Let M be the foot of the perpendicular from C to AB. Prove that lines AQ, BP and CM are concurrent if and only if $\angle BCQ = \angle ACP$.

471. Let I and O denote the incentre and the circumcentre, respectively, of triangle ABC. Assume that triangle ABC is not equilateral. Prove that $\angle AIO \leq 90°$ if and only if $2BC \leq AB + CA$, with equality holding only simultaneously.