PROBLEMS FOR FEBRUARY

Please send your solution to
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no later than March 31, 2004. It is important that your complete mailing address and your email address appear on the front page.

290. The School of Architecture in the *Olymon* University proposed two projects for the new Housing Campus of the University. In each project, the campus is designed to have several identical dormitory buildings, with the same number of one-bedroom apartments in each building. In the first project, there are 12096 apartments in total. There are eight more buildings in the second project than in the first, and each building has more apartments, which raises the total of apartments in the project to 23625. How many buildings does the second project require?

291. The n-sided polygon $A_1, A_2, \cdots, A_n$ $(n \geq 4)$ has the following property: The diagonals from each of its vertices divide the respective angle of the polygon into $n-2$ equal angles. Find all natural numbers n for which this implies that the polygon $A_1A_2 \cdots A_n$ is regular.

292. 1200 different points are randomly chosen on the circumference of a circle with centre O. Prove that it is possible to find two points on the circumference, M and N, so that:

- M and N are different from the chosen 1200 points;
- $\angle MON = 30°$;
- there are *exactly* 100 of the 1200 points inside the angle MON.

293. Two players, Amanda and Brenda, play the following game: Given a number n, Amanda writes n different natural numbers. Then, Brenda is allowed to erase several (including none, but not all) of them, and to write either $+$ or $-$ in front of each of the remaining numbers, making them positive or negative, respectively, Then they calculate their sum. Brenda wins the game is the sum is a multiple of 2004. Otherwise the winner is Amanda. Determine which one of them has a winning strategy, for the different choices of n. Indicate your reasoning and describe the strategy.

294. The number $N = 10101 \cdots 0101$ is written using $n+1$ ones and n zeros. What is the least possible value of n for which the number N is a multiple of 9999?

295. In a triangle ABC, the angle bisectors AM and CK (with M and K on BC and AB respectively) intersect at the point O. It is known that

$$|AO| \div |OM| = \frac{\sqrt{6}+\sqrt{3}+1}{2}$$

and

$$|CO| \div |OK| = \frac{\sqrt{2}}{\sqrt{3}-1} .$$

Find the measures of the angles in triangle ABC.

296. Solve the equation

$$5\sin x + \frac{5}{2\sin x} - 5 = 2\sin^2 x + \frac{1}{2\sin^2 x} .$$