PROBLEMS FOR SEPTEMBER

Please send your solution to
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no later than October 15, 2003. It is important that your complete mailing address and your email address appear on the front page.

255. Prove that there is no positive integer that, when written to base 10, is equal to its kth multiple when its initial digit (on the left) is transferred to the right (units end), where $2 \leq k \leq 9$ and $k \neq 3$.

256. Find the condition that must be satisfied by y_1, y_2, y_3, y_4 in order that the following set of six simultaneous equations in x_1, x_2, x_3, x_4 is solvable. Where possible, find the solution.

$$x_1 + x_2 = y_1y_2 \qquad x_1 + x_3 = y_1y_3 \qquad x_1 + x_4 = y_1y_4$$
$$x_2 + x_3 = y_2y_3 \qquad x_2 + x_4 = y_2y_4 \qquad x_3 + x_4 = y_3y_4 \ .$$

257. Let n be a positive integer exceeding 1. Discuss the solution of the system of equations:

$$ax_1 + x_2 + \cdots + x_n = 1$$
$$x_1 + ax_2 + \cdots + x_n = a$$
$$\cdots$$
$$x_1 + x_2 + \cdots + ax_i + \cdots + x_n = a^{i-1}$$
$$\cdots$$
$$x_1 + x_2 + \cdots + x_i + \cdots + ax_n = a^{n-1} \ .$$

258. The infinite sequence $\{a_n; n = 0, 1, 2, \cdots\}$ satisfies the recursion

$$a_{n+1} = a_n^2 + (a_n - 1)^2$$

for $n \geq 0$. Find all rational numbers a_0 such that there are four distinct indices p, q, r, s for which $a_p - a_q = a_r - a_s$.

259. Let ABC be a given triangle and let $A'BC$, $AB'C$, ABC' be equilateral triangles erected outwards on the sides of triangle ABC. Let Ω be the circumcircle of $A'B'C'$ and let A'', B'', C'' be the respective intersections of Ω with the lines AA', BB', CC'.

Prove that AA', BB', CC' are concurrent and that

$$AA'' + BB'' + CC'' = AA' = BB' = CC' \ .$$

260. $TABC$ is a tetrahedron with volume 1, G is the centroid of triangle ABC and O is the midpoint of TG. Reflect $TABC$ in O to get $T'A'B'C'$. Find the volume of the intersection of $TABC$ and $T'A'B'C'$.

261. Let $x, y, z > 0$. Prove that

$$\frac{x}{x + \sqrt{(x+y)(x+z)}} + \frac{y}{y + \sqrt{(x+y)(y+z)}} + \frac{z}{z + \sqrt{(x+z)(y+z)}} \leq 1 \ .$$