PROBLEMS FOR OCTOBER

Please send your solution to
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no later than November 30, 2003. It is important that your complete mailing address and your email address appear on the front page.

262. Let ABC be an acute triangle. Suppose that P and U are points on the side BC so that P lies between B and U, that Q and V are points on the side CA so that Q lies between C and V, and that R and W are points on the side AB so that R lies between A and W. Suppose also that

$$\angle APU = \angle AUP = \angle BQV = \angle BVQ = \angle CRW = \angle CWR .$$

The lines AP, BQ and CR bound a triangle T_1 and the lines AU, BV and CW bound a triangle T_2. Prove that all six vertices of the triangles T_1 and T_2 lie on a common circle.

263. The ten digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ are each used exactly once altogether to form three positive integers for which the largest is the sum of the other two. What are the largest and the smallest possible values of the sum?

264. For the real parameter a, solve for real x the equation

$$x = \sqrt{a + \sqrt{a + x}} .$$

A complete answer will discuss the circumstances under which a solution is feasible.

265. Note that $959^2 = 919681$, $919 + 681 = 40^2$; $960^2 = 921600$, $921 + 600 = 39^2$; and $961^2 = 923521$, $923 + 521 = 38^2$. Establish a general result of which these are special instances.

266. Prove that, for any positive integer n, $\binom{2n}{n}$ divides the least common multiple of the numbers $1, 2, 3, \cdots, 2n - 1, 2n$.

267. A non-orthogonal reflection in an axis a takes each point on a to itself, and each point P not on a to a point P' on the other side of a in such a way that a intersects PP' at its midpoint and PP' always makes a fixed angle θ with a. Does this transformation preserves lines? preserve angles? Discuss the image of a circle under such a transformation.

268. Determine all continuous real functions f of a real variable for which

$$f(x + 2f(y)) = f(x) + y + f(y)$$

for all real x and y.