PROBLEMS FOR JUNE

Please send your solution to
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no later than August 10, 2003. It is important that your complete mailing address and your email address appear on the front page.

234. A square of side length 100 is divided into 10000 smaller unit squares. Two squares sharing a common side are called *neighbours*.

 (a) Is it possible to colour an even number of squares so that each coloured square has an even number of coloured neighbours?

 (b) Is it possible to colour an odd number of squares so that each coloured square has an odd number of coloured neighbours?

235. Find all positive integers, N, for which:

 (i) N has exactly sixteen positive divisors: $1 = d_1 < d_2 < \cdots < d_{16} = N$;

 (ii) the divisor with the *index* d_5 (namely, d_{d_5}) is equal to $(d_2 + d_4) \times d_6$ (the product of the two).

236. For any positive real numbers a, b, c, prove that

$$\frac{1}{b(a+b)} + \frac{1}{c(b+c)} + \frac{1}{a(c+a)} \geq \frac{27}{2(a+b+c)^2} .$$

237. The sequence $\{a_n : n = 1, 2, \cdots\}$ is defined by the recursion

$$a_1 = 20 \qquad a_2 = 30$$

$$a_{n+2} = 3a_{n+1} - a_n \qquad \text{for } \ n \geq 1 .$$

 Find all natural numbers n for which $1 + 5a_n a_{n+1}$ is a perfect square.

238. Let ABC be an acute-angled triangle, and let M be a point on the side AC and N a point on the side BC. The circumcircles of triangles CAN and BCM intersect at the two points C and D. Prove that the line CD passes through the circumcentre of triangle ABC if and only if the right bisector of AB passes through the midpoint of MN.

239. Find all natural numbers n for which the diophantine equation

$$(x + y + z)^2 = nxyz$$

 has positive integer solutions x, y, z.

240. In a competition, 8 judges rate each contestant “yes” or “no”. After the competition, it turned out, that for any two contestants, two judges marked the first one by “yes” and the second one also by “yes”; two judges have marked the first one by “yes” and the second one by “no”; two judges have marked the first one by “no” and the second one by “yes”; and, finally, two judges have marked the first one by “no” and the second one by “no”. What is the greatest number of contestants?