PROBLEMS FOR SEPTEMBER

Solutions should be submitted to
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Solution to these problems should be postmarked no later than **October 31, 2000**.

31. Let x, y, z be positive real numbers for which $x^2 + y^2 + z^2 = 1$. Find the minimum value of

$$S = \frac{xy}{z} + \frac{yz}{x} + \frac{zx}{y} \ .$$

32. The segments BE and CF are altitudes of the acute triangle ABC, where E and F are points on the segments AC anbd AB, resp[ectively. ABC is inscribed in the circle **Q** with centre O. Denote the orthocentre of ABC be H, and the midpoints of BC and AH be M and K, respectively. Let $\angle CAB = 45°$.

    (a) Prove, that the quadrilateral $MEKF$ is a square.

    (b) Prove that the midpioint of both diagonals of $MEKF$ is also the midpoint of the segment OH.

    (c) Find the length of EF, if the radius of **Q** has length 1 unit.

33. Prove the inequality $a^2 + b^2 + c^2 + 2abc < 2$, if the numbers a, b, c are the lengths of the sides of a triangle with perimeter 2.

34. Each of the edges of a cube is 1 unit in length, and is divided by two points into three equal parts. Denote by **K** the solid with vertices at these points.

    (a) Find the volume of **K**.

    (b) Every pair of vertices of **K** is connected by a segment. Some of the segments are coloured. Prove that it is always possible to find two vertices which are endpoints of the same number of coloured segments.

35. There are n points on a circle whose radius is 1 unit. What is the greatest number of segments between two of them, whose length exceeds $\sqrt{3}$?

36. Prove that there are not three rational numbers x, y, z such that

$$x^2 + y^2 + z^2 + 3(x + y + z) + 5 = 0 \ .$$