PROBLEMS FOR JULY

Solutions should be submitted to
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Solution to these problems should be postmarked no later than **August 30, 2000**.

Notes: An *acute triangle* has all of its angles less than 90°. The *orthocentre* of a triangle is the intersection point of its altitudes. Points are *collinear* iff they lie on a straight line.

19. Is it possible to divide the natural numbers $1, 2, \cdots, n$ into two groups, such that the squares of the members in each group have the same sum, if (a) $n = 40000$; (b) $n = 40002$? Explain your answer.

20. Given any six irrational numbers, prove that there are always three of them, say a, b, c, for which $a + b$, $b + c$ and $c + a$ are irrational.

21. The natural numbers x_1, x_2, $\cdots$, x_{100} are such that

$$\frac{1}{\sqrt{x_1}} + \frac{1}{\sqrt{x_2}} + \cdots + \frac{1}{\sqrt{x_{100}}} = 20\ .$$

Prove that at least two of the numbers are equal.

22. Let $\mathbf{R}$ be a rectangle with dimensions 11×12. Find the least natural number n for which it is possible to cover $\mathbf{R}$ with n rectangles, each of size 1×6 or 1×7, with no two of these having a common interior point.

23. Given 21 points on the circumference of a circle, prove that at least 100 of the arcs determined by pairs of these points subtend an angle not exceeding 120° at the centre.

24. ABC is an acute triangle with orthocentre H. Denote by M and N the midpoints of the respective segments AB and CH, and by P the intersection point of the bisectors of angles CAH and CBH. Prove that the points M, N and P are collinear.