

PROBLEMS FOR APRIL

Please send your solution to

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no later than May 15, 2006 It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

437. Let a, b, c be the side lengths and m_a, m_b, m_c the lengths of their respective medians, of an arbitrary triangle ABC . Show that

$$\frac{3}{4} < \frac{m_a + m_b + m_c}{a + b + c} < 1 .$$

Furthermore, show that one cannot find a smaller interval to bound the ratio.

438. Determine all sets (x, y, z) of real numbers for which

$$x + y = 2 \quad \text{and} \quad xy - z^2 = 1 .$$

439. A natural number n , less than or equal to 500, has the property that when one chooses a number m randomly among $\{1, 2, 3, \dots, 500\}$, the probability that m divides n (*i.e.*, n/m is an integer) is $1/100$. Find the largest such n .
440. You are to choose 10 distinct numbers from $\{1, 2, 3, \dots, 2006\}$. Show that you can choose such numbers with a sum greater than 10039 in more ways than you can choose such numbers with a sum less than 10030.
441. Prove that, no matter how 15 points are placed inside a circle of radius 2 (including the boundary), there exists a circle of radius 1 (including the boundary) containing at least 3 of the 15 points.
442. Prove that the regular tetrahedron has minimum diameter among all tetrahedra that circumscribe a given sphere. (The diameter of a tetrahedron is the length of its longest edge.)
443. For $n \geq 3$, show that $n - 1$ straight lines are sufficient to go through the interior of every square of an $n \times n$ chessboard. Are $n - 1$ lines necessary?