PROBLEMS FOR FEBRUARY

Please send your solution to
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no later than March 31, 2003. It is important that your complete mailing address and your email address appear on the front page.

206. In a group consisting of five people, among any three people, there are two who know each other and two neither of whom knows the other. Prove that it is possible to seat the group around a circular table so that each adjacent pair knows each other.

207. Let n be a positive integer exceeding 1. Suppose that $A = (a_1, a_2, \cdots, a_m)$ is an ordered set of $m = 2^n$ numbers, each of which is equal to either 1 or -1. Let

$$S(A) = (a_1a_2, a_2a_3, \cdots, a_{m-1}a_m, a_ma_1) \ .$$

Define, $S^0(A) = A$, $S^1(A) = S(A)$, and for $k \geq 1$, $S^{k+1} = S(S^k(A))$. Is it always possible to find a positive integer r for which $S^r(A)$ consists entirely of 1s?

208. Determine all positive integers n for which $n = a^2 + b^2 + c^2 + d^2$, where $a < b < c < d$ and a, b, c, d are the four smallest positive divisors of n.

209. Determine all positive integers n for which $2^n - 1$ is a multiple of 3 and $(2^n - 1)/3$ has a multiple of the form $4m^2 + 1$ for some integer m.

210. ABC and DAC are two isosceles triangles for which B and D are on opposite sides of AC, $AB = AC$, $DA = DC$ $\angle BAC = 20°$ and $\angle ADC = 100°$. Prove that $AB = BC + CD$.

211. Let ABC be a triangle and let M be an interior point. Prove that

$$\min \{MA, MB, MC\} + MA + MB + MC < AB + BC + CA \ .$$

212. A set S of points in space has at least three elements and satisfies the condition that, for any two distinct points A and B in S, the right bisecting plane of the segment AB is a plane of symmetry for S. Determine all possible finite sets S that satisfy the condition.