## PROBLEMS FOR OCTOBER

Please send your solutions to
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109. Suppose that

$$\frac{x^2+y^2}{x^2-y^2}+\frac{x^2-y^2}{x^2+y^2}=k\ .$$

Find, in terms of $k$, the value of the expression

$$\frac{x^8+y^8}{x^8-y^8}+\frac{x^8-y^8}{x^8+y^8}\ .$$

110. Given a triangle $ABC$ with an area of 1. Let $n>1$ be a natural number. Suppose that $M$ is a point on the side $AB$ with $AB=nAM$, $N$ is a point on the side $BC$ with $BC=nBN$, and $Q$ is a point on the side $CA$ with $CA=nCQ$. Suppose also that $\{T\}=AN\cap CM$, $\{R\}=BQ\cap AN$ and $\{S\}=CM\cap BQ$, where $\cap$ signifies that the singleton is the intersection of the indicated segments. Find the area of the triangle $TRS$ in terms of $n$.

111. (a) Are there four different numbers, not exceeding 10, for which the sum of any three is a prime number?

(b) Are there five different natural numbers such that the sum of every three of them is a prime number?

112. Suppose that the measure of angle $BAC$ in the triangle $ABC$ is equal to $\alpha$. A line passing through the vertex $A$ is perpendicular to the angle bisector of $\angle BAC$ and intersects the line $BC$ at the point $M$. Find the other two angles of the triangle $ABC$ in terms of $\alpha$, if it is known that $BM=BA+AC$.

113. Find a function that satisfies all of the following conditions:

(a) $f$ is defined for every positive integer $n$;

(b) $f$ takes only positive values;

(c) $f(4)=4$;

(d)

$$\frac{1}{f(1)f(2)}+\frac{1}{f(2)f(3)}+\cdots+\frac{1}{f(n)f(n+1)}=\frac{f(n)}{f(n+1)}\ .$$

114. A natural number is a multiple of 17. Its binary representation (*i.e.*, when written to base 2) contains exactly three digits equal to 1 and some zeros.

(a) Prove that there are at least six digits equal to 0 in its binary representation.

(b) Prove that, if there are exactly seven digits equal to 0 and three digits equal to 1, then the number must be even.