Canadian Mathematical Society

2020 Canadian Open Mathematics Challenge

A competition of the Canadian Mathematical Society and supported by the Actuarial Profession.

CMS SMC

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Student Identification

Please print clearly and complete all information below. Failure to print legibly or provide complete information may result in your exam being disqualified. This exam is not considered valid unless it is accompanied by your test supervisor's signed declaration.

1. Given/First Name: (Required)

2. Sur-/Last Name: (Required)

3. Are you currently registered in full-time attendance at an elementary, secondary or Cégep school, or home schooled and have been since at least September 15th of this year? (Required for qualification)

☐ **Yes** ☐ No

4. Are you a Canadian Citizen or a Permanent Resident of Canada (regardless of your current address)? (Required for qualification)

☐ **Yes** ☐ No

5. Date of Birth: (Required)

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6. Grade: (Required)

☐ 8 ☐ 10 ☐ 12

☐ 9 ☐ 11 ☐ Cégep

other: ____________

7. Gender: (Optional)

☐ Female

☐ Male

8. Email Address: (Optional)

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Each page of each copy is uniquely pre-coded to facilitate computer-assisted marking.

Question A1 (4 points)

At a party, if each kid took one apple from the fruit bucket then 7 apples would still remain in the bucket. However, if each kid had an appetite for two apples, the supply would be 16 apples short. How many kids were at the party?

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question A2 (4 points)

It is possible to create 24 distinct four digit numbers, where each number uses each of the digits 1, 2, 3, and 4 exactly once. How many of these are divisible by 4?

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question A3 (4 points)

One person can make n objects in an hour. Ann begins at 10:00 AM, Bob begins at 10:20 AM, then Cody and Deb begin at 10:40 AM. Working at the same constant speed, by 11:00 AM, they have made 28 objects. What is n?

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question A4 (4 points)

If a, b are the roots of the polynomial $x^2 + x - 2020 = 0$, find $a^2 - b$.

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B1 (6 points)

The set S contains 6 distinct positive integers. The average of the two smallest numbers is 5 and the average of the two largest is 22. What is the greatest possible average of all the numbers in the set S?

Your solution:

Your final answer:

[A correct answer here earns full marks]

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Question B2 (6 points)

Alice places a coin, heads up, on a table then turns off the light and leaves the room. Bill enters the room with 2 coins and flips them onto the table and leaves. Carl enters the room, in the dark, and removes a coin at random. Alice reenters the room, turns on the light and notices that both coins are heads. What is the probability that the coin Carl removed was also heads?

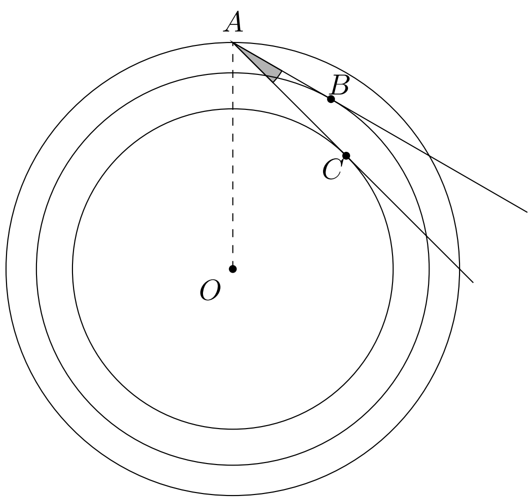
Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B3 (6 points)

Three circles with the same centre O and areas 2π, 3π and 4π are drawn. From a point A on the largest circle, tangent lines are drawn to points B on the middle circle and C on the smallest circle. If B, C are on the same side of OA, find the exact value of $\angle BAC$.



Your solution:

Your final answer:

[A correct answer here earns full marks]

Question B4 (6 points)

An ant walks from the bottom left corner of a 10×10 square grid to the diagonally-opposite corner, always walking along grid lines and taking as short a route as possible. Let $N(k)$ be the number of different paths that ant could follow if it makes exactly k turns. Find $N(6) - N(5)$.

Your solution:

Your final answer:

[A correct answer here earns full marks]

Question C1 (10 points)

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Find the areas of the three polygons described by the following conditions (a), (b) and (c), respectively.

(a) the system of inequalities $|x| \leq 1$ and $|y| \leq 1$

(b) the inequality $|x| + |y| \leq 10$

(c) the inequality $|x| + |y| + |x + y| \leq 2020$

Your solution:

You **must** show all your work.

Question C2 (10 points)

An expression like

$$x = 1 + \frac{1}{2 + \frac{1}{3+\frac{1}{4}}}$$

is called a continued fraction.

(a) Write x given above as a reduced fraction of the form $\frac{a}{b}$ where a and b are positive integers.

(b) Write $\dfrac{355}{113}$ as a continued fraction in the form $a+\frac{1}{b+\frac{1}{c}}$, where a, b, c are positive integers.

(c) Let

$$y = 8 + \frac{1}{8 + \frac{1}{8+\frac{1}{8+\cdots}}}$$

where the process continues indefinitely. Given that y can be written in the form $p + \sqrt{q}$, find the integers p and q.

Your solution:

You **must** show all your work.

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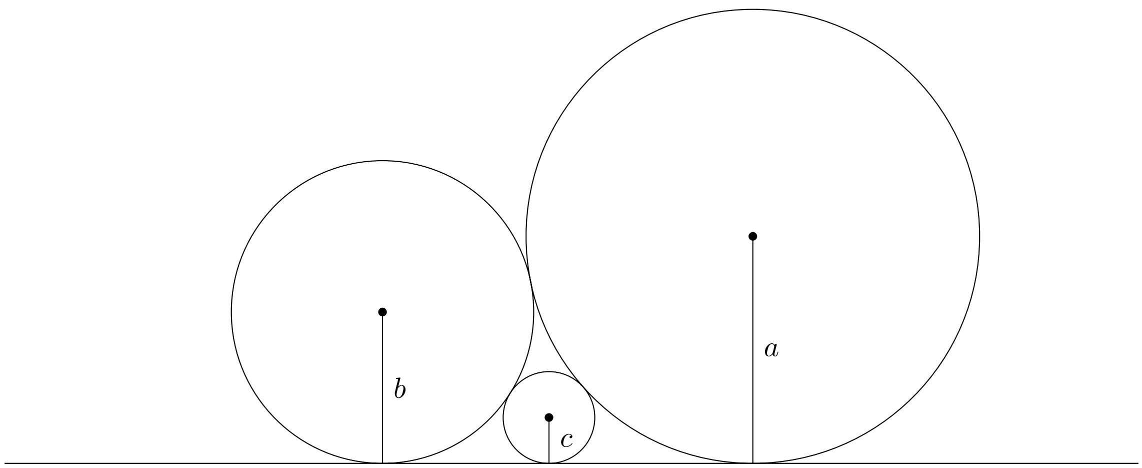
Question C3 (10 points)

Three circles with radii $0 < c < b < a$ are tangent to each other and to a line as shown in the diagram below.



(a) Let radii $a = 16$ and $b = 4$. Find the distance d between the points of tangency of these two circles to the line.

(b) Let radii $a = 16$ and $b = 4$. Find the radius c.

(c) The configuration is called *nice* if a, b, and c are all integers. Among all nice configurations, find the smallest possible value of c.

Your solution:

You **must** show all your work.

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Question C4 (10 points)

Let $S = \{4, 8, 9, 16, \ldots\}$ be the set of integers of the form m^k for integers $m, k \geq 2$. For a positive integer n, let $f(n)$ denote the number of ways to write n as the sum of (one or more) *distinct* elements of S. For example, $f(5) = 0$ since there are no ways to express 5 in this fashion, and $f(17) = 1$ since $17 = 8 + 9$ is the only way to express 17.

(a) Prove that $f(30) = 0$.

(b) Show that $f(n) \geq 1$ for $n \geq 31$.

(c) Let T be the set of integers for which $f(n) = 3$. Prove that T is finite and non-empty, and find the largest element of T.

Your solution:

You **must** show all your work.

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