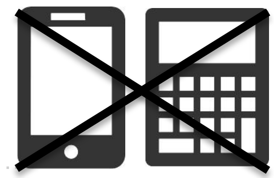
The Sun Life Financial
Canadian Open Mathematics Challenge
November 5/6, 2015

STUDENT INSTRUCTION SHEET

General Instructions



Mobile phones and calculators are NOT permitted.

1) Do not open the exam booklet until instructed to do so by your supervising teacher.
2) The supervisor will give you **five minutes before the exam starts** to fill in the identification section on the exam cover sheet. You don't need to rush. Be sure to fill in all information fields and print legibly.
3) Once you have completed the exam and given it to your supervising teacher you may leave the exam room.
4) The contents of the COMC 2015 exam and your answers and solutions must not be publicly discussed (including online) for at least 24 hours.

Exam Format

You have 2 hours and 30 minutes to complete the COMC. There are three sections to the exam:

PART A: Four introductory questions worth 4 marks each. Partial marks may be awarded for work shown.

PART B: Four more challenging questions worth 6 marks each. Partial marks may be awarded for work shown.

PART C: Four long-form proof problems worth 10 marks each. Complete work must be shown. Partial marks may be awarded.

Diagrams are not drawn to scale; they are intended as aids only.

Work and Answers

All solution work and answers are to be presented in this booklet in the boxes provided – do not include additional sheets. Marks are awarded for completeness and clarity. For sections A and B, it is not necessary to show your work in order to receive full marks. However, if your answer or solution is incorrect, any work that you do and present in this booklet will be considered for partial marks. For section C, you must **show** your work and provide the correct answer or solution to receive full marks.

It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2 + \sqrt{7}$, etc., rather than as 12.566, 4.646, etc. The names of all award winners will be published on the Canadian Mathematical Society web site https://cms.math.ca/comc.

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Please print clearly and ***complete all information below****. Failure to print legibly or provide complete information may result in your exam being disqualified. This exam is not considered valid unless it is accompanied by your test supervisor's signed form.*

First Name:

Last Name:

Are you currently registered in full-time attendance at an elementary, secondary or Cégep school, or home schooled and have been since Sept. 15th of this year?

☐ Yes ☐ No

Are you a Canadian Citizen or a Permanent Resident of Canada (regardless of current address)?

☐ Yes ☐ No

Grade:

☐ 8 ☐ 9 ☐ 10
☐ 11 ☐ 12 ☐ Cégep
Other: ________

T-Shirt Size:

(Optional. For prize draw)

☐ S ☐ M
☐ L ☐ XL ☐ XXL

Date of Birth:

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Gender: (Optional)

☐ Male ☐ Female

Email Address:

For official use only:

A1	A2	A3	A4

B1	B2	B3	B4

C1	C2	C3	C4

TOTAL

_______ Marker initials _______ Data entry initials _______ Verification initials

Part A: Question 1 (4 marks)

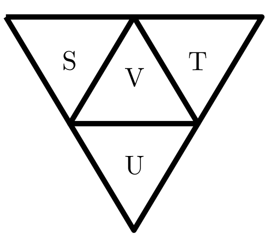
A palindrome is a number where the digits read the same forwards or backwards, such as 4774 or 505. What is the smallest palindrome that is larger than 2015?

Your Solution:

Your final answer:

Part A: Question 2 (4 marks)

In the picture below, there are four triangles labelled S, T, U, and V. Two of the triangles will be coloured red and the other two triangles will be coloured blue. How many ways can the triangles be coloured such that the two blue triangles have a common side?

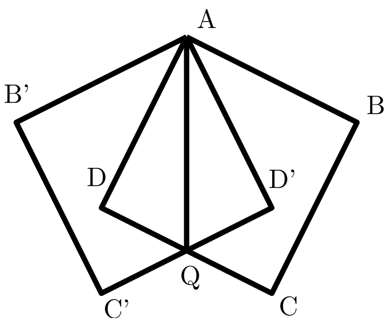


Your Solution:

Your final answer:

Part A: Question 3 (4 marks)

In the given figure, $ABCD$ is a square with sides of length 4, and Q is the midpoint of CD. $ABCD$ is reflected along the line AQ to give the square $AB'C'D'$. The two squares overlap in the quadrilateral $ADQD'$. Determine the area of quadrilateral $ADQD'$.



Your Solution:

Your final answer:

Part A: Question 4 (4 marks)

The area of a rectangle is 180 units^2 and the perimeter is 54 units. If the length of each side of the rectangle is increased by six units, what is the area of the resulting rectangle?

Your Solution:

Your final answer:

Part B: Question 1 (6 marks)

Given an integer $n \geq 2$, let $f(n)$ be the second largest positive divisor of n. For example, $f(12) = 6$ and $f(13) = 1$. Determine the largest positive integer n such that $f(n) = 35$.

Your Solution:

Your final answer:

Part B: Question 2 (6 marks)

Let ABC be a right triangle with $\angle BCA = 90^o$. A circle with diameter AC intersects the hypotenuse AB at K. If $BK : AK = 1 : 3$, find the measure of the angle $\angle BAC$.

Your Solution:

Your final answer:

Part B: Question 3 (6 marks)

An arithmetic sequence is a sequence where each term after the first is the sum of the previous term plus a constant value. For example, $3, 7, 11, 15, \ldots$ is an arithmetic sequence.

S is a sequence which has the following properties:

- The first term of S is positive.
- The first three terms of S form an arithmetic sequence.
- If a square is constructed with area equal to a term in S, then the perimeter of that square is the next term in S.

Determine all possible values for the third term of S.

Your Solution:

Your final answer:

Part B: Question 4 (6 marks)

A farmer has a flock of n sheep, where $2000 \leq n \leq 2100$. The farmer puts some number of the sheep into one barn and the rest of the sheep into a second barn. The farmer realizes that if she were to select two different sheep at random from her flock, the probability that they are in different barns is exactly $\frac{1}{2}$. Determine the value of n.

Your Solution:

Your final answer:

Part C: Question 1 (10 marks)

A quadratic polynomial $f(x) = x^2 + px + q$, with p and q real numbers, is said to be a *double-up polynomial* if it has two real roots, one of which is twice the other.

(a) If a double-up polynomial $f(x)$ has $p = -15$, determine the value of q.

(b) If $f(x)$ is a double-up polynomial with one of the roots equal to 4, determine all possible values of $p + q$.

(c) Determine all double-up polynomials for which $p + q = 9$.

Your solution:

Part C: Question 2 (10 marks)

Let $O = (0, 0), Q = (13, 4), A = (a, a), B = (b, 0)$, where a and b are positive real numbers with $b \geq a$. The point Q is on the line segment AB.

(a) Determine the values of a and b for which Q is the midpoint of AB.

(b) Determine all values of a and b for which Q is on the line segment AB and the triangle OAB is isosceles and right-angled.

(c) There are infinitely many line segments AB that contain the point Q. For how many of these line segments are a and b both integers?

Your solution:

Part C: Question 3 (10 marks)

(a) If $n = 3$, determine all integer values of m such that $m^2 + n^2 + 1$ is divisible by $m - n + 1$ and $m + n + 1$.

(b) Show that for any integer n there is always at least one integer value of m for which $m^2 + n^2 + 1$ is divisible by both $m - n + 1$ and $m + n + 1$.

(c) Show that for any integer n there are only a finite number of integer values m for which $m^2 + n^2 + 1$ is divisible by both $m - n + 1$ and $m + n + 1$.

Your solution:

Part C: Question 4 (10 marks)

Mr. Whitlock is playing a game with his math class to teach them about money. Mr. Whitlock's math class consists of $n \geq 2$ students, whom he has numbered from 1 to n. Mr. Whitlock gives $m_i \geq 0$ dollars to student i, for each $1 \leq i \leq n$, where each m_i is an integer and $m_1 + m_2 + \cdots + m_n \geq 1$.

We say a student is a *giver* if no other student has more money than they do and we say a student is a *receiver* if no other student has less money than they do. To play the game, each student who is a giver, gives one dollar to each student who is a receiver (it is possible for a student to have a negative amount of money after doing so). This process is repeated until either all students have the same amount of money, or the students reach a distribution of money that they had previously reached.

(a) Give values of $n, m_1, m_2, \ldots, m_n$ for which the game ends with at least one student having a negative amount of money, and show that the game does indeed end this way.

(b) Suppose there are n students. Determine the smallest possible value k_n such that if $m_1 + m_2 + \cdots + m_n \geq k_n$ then no player will ever have a negative amount of money.

(c) Suppose $n = 5$. Determine all quintuples $(m_1, m_2, m_3, m_4, m_5)$, with $m_1 \leq m_2 \leq m_3 \leq m_4 \leq m_5$, for which the game ends with all students having the same amount of money.

Your solution:

Canadian Mathematical Society
Société mathématique du Canada

Sun Life Financial
Canadian Open Mathematics Challenge

Maplesoft
NELSON EDUCATION
UNIVERSITY OF TORONTO
UNIVERSITY OF CALGARY
UBC
UNIVERSITY OF SASKATCHEWAN
UNIVERSITY OF MANITOBA
DALHOUSIE UNIVERSITY
MEMORIAL UNIVERSITY
UNIVERSITY of Prince Edward ISLAND
UNB