The Canadian Mathematical Society
in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
presents the

Canadian Open Mathematics Challenge

Wednesday, November 24, 2004

Time: $2\frac{1}{2}$ hours

Calculators are NOT permitted.

Do not open this booklet until instructed to do so.
There are two parts to this paper.

PART A
This part of the paper consists of 8 questions, each worth 5 marks. You can earn full value for each question by entering the correct answer in the space provided. Any work you do in obtaining an answer will be considered for part marks if you do not have the correct answer, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B
This part of the paper consists of 4 questions, each worth 10 marks. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet.
Marks are awarded for completeness, clarity, and style of presentation. A correct solution poorly presented will not earn full marks.

NOTE: At the completion of the contest, insert the information sheet inside the answer booklet.

Canadian Open Mathematics Challenge

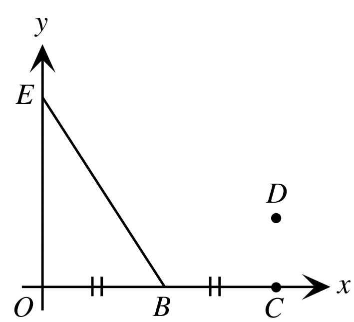
NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc.
	4.	Calculators are **not** allowed.

PART A

1. If $x + 2y = 84 = 2x + y$, what is the value of $x + y$?

2. Let S be the set of all three-digit positive integers whose digits are 3, 5 and 7, with no digit repeated in the same integer. Calculate the remainder when the sum of all of the integers in S is divided by 9.

3. In the diagram, point E has coordinates $(0, 2)$, and B lies on the positive x-axis so that $BE = \sqrt{7}$. Also, point C lies on the positive x-axis so that $BC = OB$. If point D lies in the first quadrant such that $\angle CBD = 30°$ and $\angle BCD = 90°$, what is the length of ED?



4. A function $f(x)$ has the following properties:

   i) $f(1) = 1$

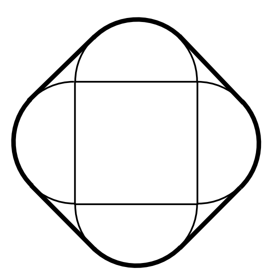
   ii) $f(2x) = 4f(x) + 6$

   iii) $f(x + 2) = f(x) + 12x + 12$

   Calculate $f(6)$.

5. The Rice Tent Company sells tents in two different sizes, large and small. Last year, the Company sold 200 tents, of which one quarter were large. The sale of the large tents produced one third of the company's income. What was the ratio of the price of a large tent to the price of a small tent?

6. In the diagram, a square of side length 2 has semicircles drawn on each side. An "elastic band" is stretched tightly around the figure. What is the length of the elastic band in this position?



7. Let a and b be real numbers, with $a > 1$ and $b > 0$.
   If $ab = a^b$ and $\dfrac{a}{b} = a^{3b}$, determine the value of a.

8. A rectangular sheet of paper, $ABCD$, has $AD = 1$ and $AB = r$, where $1 < r < 2$. The paper is folded along a line through A so that the edge AD falls onto the edge AB. Without unfolding, the paper is folded again along a line through B so that the edge CB also lies on AB. The result is a triangular piece of paper. A region of this triangle is four sheets thick. In terms of r, what is the area of this region?

PART B

1. The points $A(-8, 6)$ and $B(-6, -8)$ lie on the circle $x^2 + y^2 = 100$.

   (a) Determine the equation of the line through A and B.

   (b) Determine the equation of the perpendicular bisector of AB.

   (c) The perpendicular bisector of AB cuts the circle at two points, P in the first quadrant and Q in the third quadrant. Determine the coordinates of P and Q.

   (d) What is the length of PQ? Justify your answer.

2. (a) Determine the two values of x such that $x^2 - 4x - 12 = 0$.

   (b) Determine the *one* value of x such that $x - \sqrt{4x + 12} = 0$. Justify your answer.

   (c) Determine all real values of c such that

   $$x^2 - 4x - c - \sqrt{8x^2 - 32x - 8c} = 0$$

   has precisely two distinct real solutions for x.

3. A map shows all Beryl's Llamaburgers restaurant locations in North America. On this map, a line segment is drawn from each restaurant to the restaurant that is closest to it. Every restaurant has a unique closest neighbour. (Note that if A and B are two of the restaurants, then A may be the closest to B without B being closest to A.)

   (a) Prove that no three line segments on the map can form a triangle.

   (b) Prove that no restaurant can be connected to more than five other restaurants.

4. In a *sumac sequence*, t_1, t_2, t_3, . . ., t_m, each term is an integer greater than or equal to 0. Also, each term, starting with the third, is the difference of the preceding two terms (that is, $t_{n+2} = t_n - t_{n+1}$ for $n \geq 1$). The sequence terminates at t_m if $t_{m-1} - t_m < 0$. For example, 120, 71, 49, 22, 27 is a sumac sequence of length 5.

   (a) Find the positive integer B so that the sumac sequence 150, B, . . . has the maximum possible number of terms.

   (b) Let m be a positive integer with $m \geq 5$. Determine the number of sumac sequences of length m with $t_m \leq 2000$ and with no term divisible by 5.

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Mathematics
Challenge
(English)