*The Canadian Mathematical Society*

in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

# *The Canadian Open Mathematics Challenge*

**Wednesday, November 27, 2002**

---

**Time:** $2\frac{1}{2}$ hours



**Calculators are NOT permitted.**

Do not open this booklet until instructed to do so.
There are two parts to the paper.

PART A
This part of the paper consists of 8 questions, each worth 5 marks. You can earn full value for each question by entering the correct answer in the space provided. Any work you do in obtaining an answer will be considered for part marks if you do not have the correct answer, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B
This part of the paper consists of 4 questions, each worth 10 marks. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet.
Marks are awarded for completeness, clarity, and style of presentation. A correct solution poorly presented will not earn full marks.

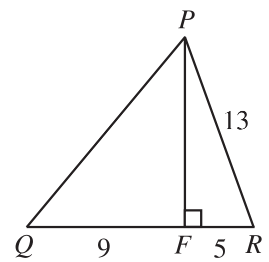
**NOTE: At the completion of the contest, insert the information sheet inside the answer booklet.**

# Canadian Open Mathematics Challenge

NOTE: 1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. It is expected that all calculations and answers will be expressed as exact numbers such as $4\pi, 2+\sqrt{7}$, etc.
4. Calculators are **not** allowed.

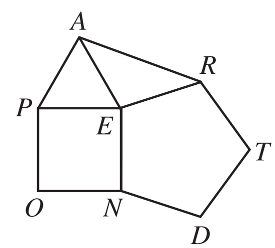
## PART A

1. In triangle $PQR$, $F$ is the point on $QR$ so that $PF$ is perpendicular to $QR$. If $PR = 13$, $RF = 5$, and $FQ = 9$, what is the perimeter of $\Delta PQR$?




2. If $x + y = 4$ and $xy = -12$, what is the value of $x^2 + 5xy + y^2$?

3. A regular pentagon is a five-sided figure which has all of its angles equal and all of its side lengths equal. In the diagram, *TREND* is a regular pentagon, *PEA* is an equilateral triangle, and *OPEN* is a square. Determine the size of $\angle EAR$.

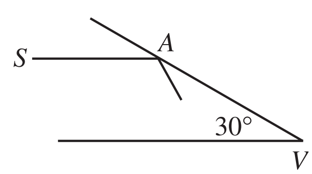



4. In a sequence of numbers, the *sum* of the first $n$ terms is equal to $5n^2 + 6n$. What is the sum of the 3rd, 4th and 5th terms in the original sequence?

5. If $m$ and $n$ are non-negative integers with $m < n$, we define $m\nabla n$ to be the sum of the integers from $m$ to $n$, including $m$ and $n$. For example, $5\nabla 8 = 5+6+7+8 = 26$.

   For every positive integer $a$, the numerical value of $\dfrac{[(2a-1)\nabla(2a+1)]}{[(a-1)\nabla(a+1)]}$ is the same. Determine this value.

6. Two mirrors meet at an angle of $30^\circ$ at the point $V$. A beam of light, from a source $S$, travels parallel to one mirror and strikes the other mirror at point $A$, as shown. After a number of reflections, the beam comes back to $S$. If $SA$ and $AV$ are both 1 metre in length, determine the total distance travelled by the beam.




7. $N$ is a five-digit positive integer. A six-digit integer $P$ is constructed by placing a 1 at the right-hand end of $N$. A second six-digit integer $Q$ is constructed by placing a 1 at the left-hand end of $N$. If $P$ is three times $Q$, determine the value of $N$.

8. Suppose that $M$ is an integer with the property that if $x$ is randomly chosen from the set $\{1,2,3,\ldots,999,1000\}$, the probability that $x$ is a divisor of $M$ is $\frac{1}{100}$. If $M \leq 1000$, determine the maximum possible value of $M$.

## PART B

1. Square $ABCD$ has vertices $A(0,0)$, $B(0,8)$, $C(8,8)$, and $D(8,0)$. The points $P(0,5)$ and $Q(0,3)$ are on side $AB$, and the point $F(8,1)$ is on side $CD$.
   (a) What is the equation of the line through $Q$ parallel to the line through $P$ and $F$?
   (b) If the line from part (a) intersects $AD$ at the point $G$, what is the equation of the line through $F$ and $G$?
   (c) The centre of the square is the point $H(4,4)$. Determine the equation of the line through $H$ perpendicular to $FG$.
   (d) A circle is drawn with centre $H$ that is tangent to the four sides of the square. Does this circle intersect the line through $F$ and $G$? Justify your answer. (A sketch is *not* sufficient justification.)

2. (a) Let $A$ and $B$ be digits (that is, $A$ and $B$ are integers between 0 and 9 inclusive). If the product of the three-digit integers $2A5$ and $13B$ is divisible by 36, determine with justification the *four* possible ordered pairs $(A,B)$.
   (b) An integer $n$ is said to be a multiple of 7 if $n = 7k$ for some integer $k$.
      (i) If $a$ and $b$ are integers and $10a + b = 7m$ for some integer $m$, prove that $a - 2b$ is a multiple of 7.
      (ii) If $c$ and $d$ are integers and $5c + 4d$ is a multiple of 7, prove that $4c - d$ is also a multiple of 7.

3. There are some marbles in a bowl. Alphonse, Beryl and Colleen each take turns removing one or two marbles from the bowl, with Alphonse going first, then Beryl, then Colleen, then Alphonse again, and so on. The player who takes the last marble from the bowl is the loser, and the other two players are the winners.
   (a) If the game starts with 5 marbles in the bowl, can Beryl and Colleen work together and force Alphonse to lose?
   (b) The game is played again, this time starting with $N$ marbles in the bowl. For what values of $N$ can Beryl and Colleen work together and force Alphonse to lose?

4. Triangle $DEF$ is acute. Circle $C_1$ is drawn with $DF$ as its diameter and circle $C_2$ is drawn with $DE$ as its diameter. Points $Y$ and $Z$ are on $DF$ and $DE$ respectively so that $EY$ and $FZ$ are altitudes of $\Delta DEF$. $EY$ intersects $C_1$ at $P$, and $FZ$ intersects $C_2$ at $Q$. $EY$ extended intersects $C_1$ at $R$, and $FZ$ extended intersects $C_2$ at $S$. Prove that $P$, $Q$, $R$, and $S$ are concyclic points.