The Canadian Mathematical Society
in collaboration with
The Center for Education
in Mathematics and Computing

The Fourth Canadian Open Mathematics Challenge

Wednesday, November 24, 1999

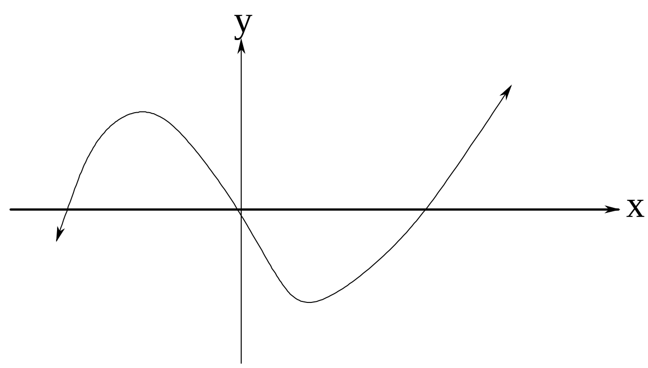
Examination Paper

Part A

Note: All questions in part A will be graded out of 5 points.

1. Determine the sum of all odd positive two-digit integers that are divisible by 5.

2. A rough *sketch* of the graph of $y = (2x+4)(x^2-3x)$ is shown. For what values of x is $y \geq 0$?



3. Solve for x : $\left(\frac{4}{9}\right)^x\left(\frac{8}{27}\right)^{1-x} = \frac{2}{3}$.

4. Solve the system of equations for x.

$$x + 2y - z = 5$$

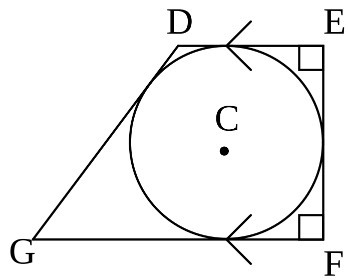
$$3x + 2y + z = 11$$

$$(x+2y)^2 - z^2 = 15$$

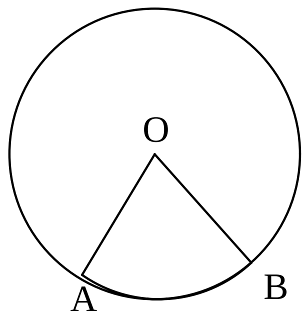
5. Determine all x which satisfy:

$$2\sin^3 x + 6\sin^2 x - \sin x - 3 = 0, 0 < x < 2\pi$$

6. A trapezoid, $DEFG$, is circumscribed about a circle that has centre C and radius 2, as is shown. The shorter of the two parallel sides, DE, has length 3 and angles DEF and EFG are right angles. Determine the area of the trapezoid.



7. The sector OAB of a circle, with centre O, has a perimeter of 12. Determine the radius of the circle which maximizes the area of the sector.



8. Find the smallest positive integer k so that the expression $\frac{14k+17}{k-9}$ becomes a fraction in the form $\frac{pd}{qd}$ where p, q and d are positive integers, p and q have no common divisors, and neither q nor d equals 1.

Part B

Note: All questions in part B will be graded out of 10 points.

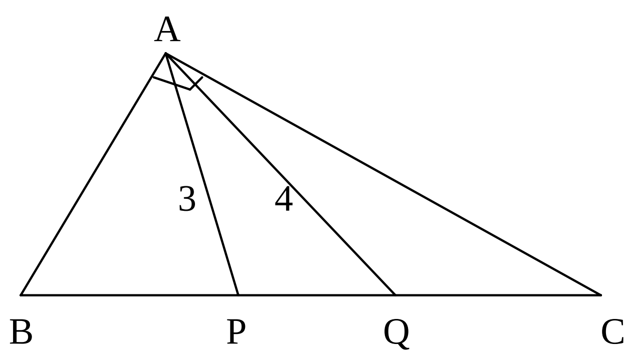
1.

(a) Two identical triangles each have an area of 24. Their vertices are determined by the intersection of the lines with equations $y = -4$,$x = 0$ and $y = \frac{-3}{4}x + b$. Determine the two possible values for b.

(b) For either of the two given triangles, a circle can be drawn to pass through its three vertices. What is the radius of this circle?

2. If $(bd + cd)$ is an odd integer, show that the cubic polynomial $x^3 + bx^2 + cx + d$ cannot be expressed in the form $(x + r)(x^2 + px + q)$ where b, c, d, r, p and q are all integers.

3. Triangle ABC is right angled with its right angle at A. The points P and Q are on the hypotenuse BC such that $BP = PQ = QC$, $AP = 3$ and $AQ = 4$. Determine the length of each side of $\triangle ABC$.



4. Triangle ABC is any one of the set of triangles having base BC equal to a and height from A to BC equal to h, with $h < \frac{\sqrt{3}}{2}a$. P is a point inside the triangle such that the value of $\angle PAB = \angle PBA = \angle PCB = \alpha$. Show that the measure of α is the same for every triangle in the set.