The Canadian Mathematical Society

in collaboration with

The Center for Education
in Mathematics and Computing

The Second Canadian Open Mathematics Challenge

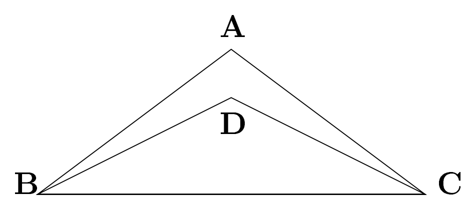
Wednesday, November 26, 1997

Examination Paper

Part A

Note: All questions in part A will be graded out of 5 points.

1. In triangle ABC, $\angle A$ equals 120 degrees. A point D is inside the triangle such that $\angle DBC = 2 \cdot \angle ABD$ and $\angle DCB = 2 \cdot \angle ACD$. Determine the measure, in degrees, of $\angle BDC$.



2. Solve the following system of equations:

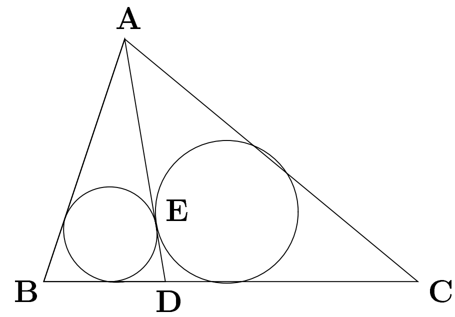
$$xy^2 = 10^8, \ \frac{x^3}{y} = 10^{10}.$$

3. Determine all points on the straight line which joins $(-4, 11)$ to $(16, -1)$ and whose coordinates are positive integers.

4. Given three distinct digits a, b and c, it is possible, by choosing two digits at a time, to form six two-digit numbers. Determine all possible sets $\{a, b, c\}$ for which the sum of the six two-digits numbers is 484.

5. Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is $\frac{1}{2}$. How many red faces are there on the second cube?

6. The triangle ABC has sides $AB = 137$, $AC = 241$, and $BC = 200$. There is a point D, on BC, such that both incircles of triangles ABD and ACD touch AD at the same point E. Determine the length of CD.



7. Determine the minimum value of $f(x)$ where

$$f(x) = (3\sin x - 4\cos x - 10)(3\sin x + 4\cos x - 10).$$

8. An hourglass is formed from two identical cones. Initially, the upper cone is filled with sand and the lower one is empty. The sand flows at a constant rate from the upper to the lower cone. It takes exactly one hour to empty the upper cone. How long does it take for the depth of sand in the lower cone to be half the depth of sand in the upper cone? (Assume that the sand stays level in both cones at all times.)

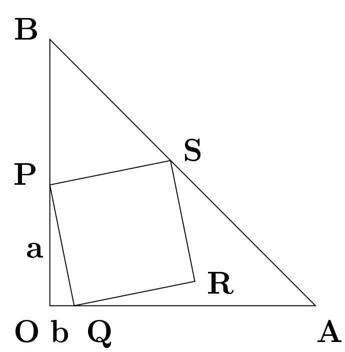
Part B

Note: All questions in part B will be graded out of 10 points.

1. The straight line l_1 with equation $x - 2y + 10 = 0$ meets the circle with equation $x^2 + y^2 = 100$ at B in the first quadrant. A line through B, perpendicular to l_1 cuts the y-axis at $P(0, t)$. Determine the value of t.

2. Consider the ten numbers $ar, ar^2, ar^3, \cdots, ar^{10}$. If their sum is 18 and the sum of their reciprocals is 6, determine their product.

3. In an isosceles right-angled triangle AOB, points P, Q and S are chosen on sides OB, OA and AB respectively such that a square $PQRS$ is formed as shown. If the lengths of OP and OQ are a and b respectively, and the area of $PQRS$ is $\frac{2}{5}$ that of triangle AOB, determine $a : b$.



4. Find all real values of x, y and z such that

$$\begin{aligned} x - \sqrt{yz} &= 42 \\ y - \sqrt{xz} &= 6 \\ z - \sqrt{xy} &= -30. \end{aligned}$$