36th Canadian Mathematical Olympiad

Wednesday, March 31, 2004

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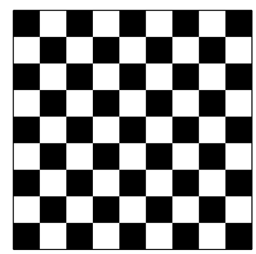
1. Find all ordered triples (x, y, z) of real numbers which satisfy the following system of equations:

$$\begin{cases} xy &= z - x - y \\ xz &= y - x - z \\ yz &= x - y - z \end{cases}$$

2. How many ways can 8 mutually non-attacking rooks be placed on the 9×9 chessboard (shown here) so that all 8 rooks are on squares of the same colour?
[Two rooks are said to be attacking each other if they are placed in the same row or column of the board.]



3. Let A, B, C, D be four points on a circle (occurring in clockwise order), with $AB < AD$ and $BC > CD$. Let the bisector of angle BAD meet the circle at X and the bisector of angle BCD meet the circle at Y. Consider the hexagon formed by these six points on the circle. If four of the six sides of the hexagon have equal length, prove that BD must be a diameter of the circle.

4. Let p be an odd prime. Prove that

$$\sum_{k=1}^{p-1} k^{2p-1} \equiv \frac{p(p+1)}{2} \pmod{p^2}.$$

[Note that $a \equiv b \pmod{m}$ means that $a - b$ is divisible by m.]

5. Let T be the set of all positive integer divisors of 2004^{100}. What is the largest possible number of elements that a subset S of T can have if no element of S is an integer multiple of any other element of S?