THE 2001 CANADIAN MATHEMATICAL OLYMPIAD

1. **Randy:** "Hi Rachel, that's an interesting quadratic equation you have written down. What are its roots?"
Rachel: "The roots are two positive integers. One of the roots is my age, and the other root is the age of my younger brother, Jimmy."
Randy: "That is very neat! Let me see if I can figure out how old you and Jimmy are. That shouldn't be too difficult since all of your coefficients are integers. By the way, I notice that the sum of the three coefficients is a prime number."
Rachel: "Interesting. Now figure out how old I am."
Randy: "Instead, I will guess your age and substitute it for x in your quadratic equation …darn, that gives me -55, and not 0."
Rachel: "Oh, leave me alone!"

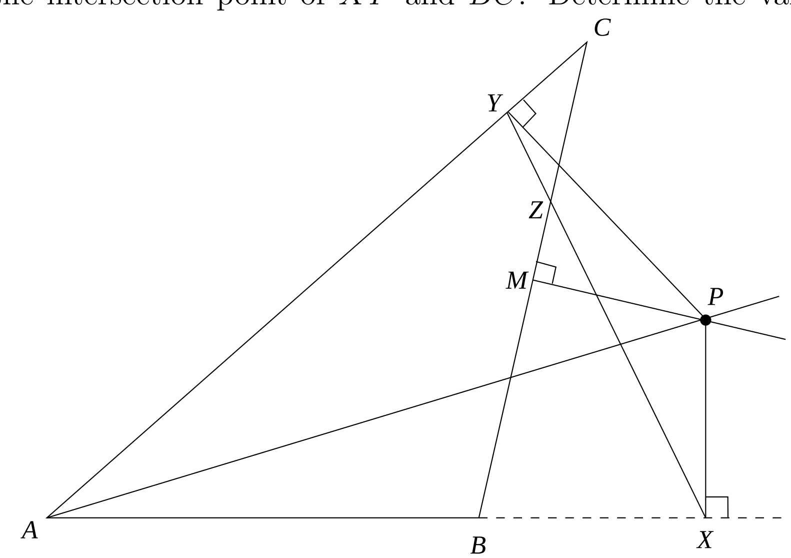
   (a) Prove that Jimmy is two years old.

   (b) Determine Rachel's age.

2. There is a board numbered -10 to 10 as shown. Each square is coloured either red or white, and the sum of the numbers on the red squares is n. Maureen starts with a token on the square labeled 0. She then tosses a fair coin ten times. Every time she flips heads, she moves the token one square to the right. Every time she flips tails, she moves the token one square to the left. At the end of the ten flips, the probability that the token finishes on a red square is a rational number of the form $\frac{a}{b}$. Given that $a + b = 2001$, determine the largest possible value for n.

-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10

3. Let ABC be a triangle with $AC > AB$. Let P be the intersection point of the perpendicular bisector of BC and the internal angle bisector of $\angle A$. Construct points X on AB (extended) and Y on AC such that PX is perpendicular to AB and PY is perpendicular to AC. Let Z be the intersection point of XY and BC. Determine the value of BZ/ZC.



4. Let n be a positive integer. Nancy is given a rectangular table in which each entry is a positive integer. She is permitted to make either of the following two moves:

(a) select a row and multiply each entry in this row by n.

(b) select a column and subtract n from each entry in this column.

Find all possible values of n for which the following statement is true:

> Given any rectangular table, it is possible for Nancy to perform a finite sequence of moves to create a table in which each entry is 0.

5. Let P_0, P_1, P_2 be three points on the circumference of a circle with radius 1, where $P_1P_2 = t < 2$. For each $i \geq 3$, define P_i to be the centre of the circumcircle of $\triangle P_{i-1}P_{i-2}P_{i-3}$.

   (a) Prove that the points $P_1, P_5, P_9, P_{13}, \ldots$ are collinear.

   (b) Let x be the distance from P_1 to P_{1001}, and let y be the distance from P_{1001} to P_{2001}. Determine all values of t for which $\sqrt[500]{x/y}$ is an integer.