

# Canadian Mathematical Olympiad

## 1991

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### PROBLEM 1

Show that the equation  $x^2 + y^5 = z^3$  has infinitely many solutions in integers  $x, y, z$  for which  $xyz \neq 0$ .

### PROBLEM 2

Let  $n$  be a fixed positive integer. Find the sum of all positive integers with the following property: In base 2, it has exactly  $2n$  digits consisting of  $n$  1's and  $n$  0's. (The first digit cannot be 0.)

### PROBLEM 3

Let  $C$  be a circle and  $P$  a given point in the plane. Each line through  $P$  which intersects  $C$  determines a chord of  $C$ . Show that the midpoints of these chords lie on a circle.

### PROBLEM 4

Ten distinct numbers from the set  $\{0, 1, 2, \dots, 13, 14\}$  are to be chosen to fill in the ten circles in the diagram. The absolute values of the differences of the two numbers joined by each segment must be different from the values for all other segments. Is it possible to do this? Justify your answer.



### PROBLEM 5

In the figure, the side length of the large equilateral triangle is 3 and  $f(3)$ , the number of parallelograms bounded by sides in the grid, is 15. For the general analogous situation, find a formula for  $f(n)$ , the number of parallelograms, for a triangle of side length  $n$ .

