ABSTRACTS / RÉSUMÉS DE CONFÉRENCES

Here are the abstracts, as of November 6, 1998. After November 6, updates to the abstracts will appear in the printed copy of the meeting programme only. Participants are asked to consult the meeting programme available during on-site registration.

Voici les résumés de conférences, en date du 6 novembre 1998. Après cette date, tout changement à cette liste ne sera apporté qu'à la copie imprimé du programme de la réunion. Les participants sont encouragés á consulter le programme, disponible á la table d'inscription.

COXETER-JAMES LECTURE / CONFÉRENCE COXETER-JAMES

HENRI DARMON, McGill University, Montreal, Quebec H3A 2K6, Canada
Recent progress in the theory of elliptic curves

After Wiles' proof of the Shimura-Taniyama conjecture, the Birch and Swinnerton-Dyer conjecture has become the outstanding open problem in the arithmetic theory of elliptic curves. In the late 80's, the work of Kolyvagin led to an almost complete proof of the Birch and Swinnerton-Dyer conjecture for elliptic curves of (analytic) rank at most one. The higher rank case remains shrouded in mystery. Recently Bertolini and I have been able to prove part of a p-adic variant of the Birch and Swinnerton Dyer conjecture which applies to curves of higher rank. The proof combines ideas that arose in the work of Wiles and earlier work of Ribet on Fermat's Last Theorem with the methods of Thaine and Kolyvagin.

DOCTORAL PRIZE LECTURE / CONFÉRENCE DU PRIX DOCTORAL

YURI BEREST, University of California, Berkeley, California 94720, USA
Lacunae for hyperbolic differential operators with variable coefficients

Lacuna of a linear hyperbolic differential operator is a domain inside its propagation cone where a proper fundamental solution vanishes identically. Huygens' principle for the classical wave equation is the simplest important example of such a phenomenon. The study of lacunas for hyperbolic equations of arbitrary order was initiated by I. G. Petrovsky (1945). Extending and clarifying his results, M. Atiyah, R. Bott and L. Gårding (1970–73) created a profound and complete theory for hyperbolic operators with constant coefficients. In contrast, much less is known about lacunas for operators with variable coefficients. The purpose of the present talk is to report on new developments in this direction. We start with a review of classical results on lacunas (related mostly to the famous Hadamard's conjecture for second order operators). We present new examples and give a solution of (a restricted version of) Hadamard's problem for wave-type operators on Minkowski spaces. In the second part of the talk we explain how these results can be extended to the case of higher order hyperbolic operators. Our goal is to give a generalization of the Petrovsky-Atiyah-Bott-Gårding theory to certain classes of partial differential operators with singular coefficients. The final part of our talk will be devoted to the discussion of various connections with algebraic geometry (structure of rings of differential operators on singular varieties), representation theory (finite reflection groups) and integrable systems.

PUBLIC LECTURE / CONFÉRENCE PUBLIQUE

H. S. M. COXETER, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada

The Descartes circle theorem and Fibonacci numbers

The *numerical distance* between two circles in the Euclidean plane is defined to be the number

$$\frac{c^2 - a^2 - b^2}{2ab},$$

where a and b are their radii while c is the ordinary distance between their centres. An infinite sequence of circles is defined to be *loxodromic* if every four consecutive members are mutually tangent. D_n denotes the numerical distance between the mth and $(m+n)$-th circles (the same for all m). Obviously $D_{-n} = D_n$. Since the numerical distance is -1 when $a = b$ and $c = 0$ so that the two circles coincide, $D_0 = -1$. Since it is 1 when $a + b = c$ so that the circles are externally tangent, $D_1 = D_2 = D_3 = 1$. Any number of further values of D_n can be determined successively by the recurrence equation

$$D_m + D_{m+4} = 2(D_{m+1} + D_{m+2} + D_{m+3}).$$

There is also an explicit formula

$$D_n = \sum_{\nu=0}^{[n/2]} \binom{n}{2\nu} f_{n-\nu-2},$$

in terms of binomial coefficients and Fibonacci numbers.

PLENARY SPEAKERS / CONFÉRENCIERS PRINCIPAUX

MIKLÓS CSÖRGŐ, Carleton University, Ottawa, Ontario K1S 5B6, Canada

Random walking around financial mathematics

The 1997 Nobel Prize in Economic Sciences was awarded to Robert C. Merton and Myron S. Scholes who, in collaboration with the late Fischer Black, developed a pioneering formula for the valuation of stock options. In 1973, Black and Scholes published what has come to be known as the Black-Scholes formula. In order to derive and properly appreciate this formula, taking a historical route, we will first review some of the fundamental notions of Brownian motion-Wiener process, as well as some elements of Itô calculus. Consequently, we will summarize the derivation of the Black-Scholes formula for the European and American options. Though the solution is in terms of geometric Brownian motion, the latter will be highlighted also in terms of geometric fractional Brownian motion. Time permitting, we will also describe some long time path properties of various geometric processes of the *call on average* (Asian) option via those of the *call on maximum* option. The latter will be based on ongoing work with Endre Csáki, Antónia Földes and Pál Révész.

ZOLTAN FÜREDI, University of Illinois at Urbana, Urbana, Illinois 61801-2975, USA

Lotto, footballpool and other covering radius problems

The aim of this talk is to review connections between Turán's (hyper)graph problem and other parts of Combinatorics, like Steiner systems, packings and coverings, constant weight codes, Kneser graphs.

The code C is called a *covering code* of X with radius r if every element of X is within Hamming distance r from at least one codeword from C. Given X we are interested in a minimum sized C. Continuing a work of Hanani, Ornstein and Sós, and Brouwer we determine the Lottery number, $L(n,k,p,2)$, the minimum number of k-subsets of an n-set such that all the $\binom{n}{p}$ p-sets are intersected by one of them in at least 2 elements, for all $n > n_0(k,p)$. Answering a question of Hämäläinen, Honkala, Litsyn and Östergard we show further connections between Turán theorem and constant weight covering codes.

WILLIAM M. GOLDMAN, University of Maryland, College Park, Maryland 20742, USA

Topology and dynamics of moduli spaces of geometric structures on surfaces

The space of representations of the fundamental group of a surface into a Lie group is a natural object which exhibits rich geometric structure and rich symmetry. In particular these moduli spaces are interesting examples of symplectic geometry. The ergodicity of the modular group acting on moduli space (for certain compact G) will be discussed, as the relationship to uniformization of geometric structures. When G is noncompact, we conjecture that there are components where the action is ergodic as well as components where the action is properly discontinuous (arising from uniformizations). Uniformizations by convex domains in the real projective plane, as well as complex hyperbolic Kleinian groups will also be discussed. Using singular hyperbolic structures, one can interpret arbitrary surface group homomorphisms in terms of uniformizations, which provides a geometric interpretation to the dynamical questions about moduli spaces.

DON O'SHEA, Mt. Holyoke College, South Hadley, Massachusetts 01075, USA
The glory and perversity of us

Mathematics has an image problem. I contend that much of that problem stems from myths that we have internalized about the discipline, the profession, and ourselves. These myths go largely unexamined and unchallenged, and often work great harm.

In particular, I talk about the notion that mathematics is inherently hierarchical, that there is a right way of teaching a topic, that mathematics is a young man's game or a sacred calling. I analyse the rather limited ways in which we talk about what we do and what we teach and ought to teach, and I study how these constrict our curricular offerings. I give examples of what seems to work and what does not, of when we excel and when we shoot ourselves in the foot.

Algebraic Geometry / Géométrie algébrique

P. Milman, Organizer

FEDOR BOGOMOLOV, Courant Institute, New York University, New York, USA
On the fundamental groups and universal coverings of complex projective surfaces and symplectic fourfolds

In the talk I will consider the problem of holomorphic convexity of the universal coverings of complex projective surfaces and the problem of representation of a finitely presented group as a fundamental group of complex projective surface or symplectic Lefschetz pencil.

ANDREW HWANG, University of Toronto
Construction of Circle-Invariant Kahler Metrics

In the total space of an Hermitian disk bundle, it is possible to construct a Kahler metric from a single function of one variable by using an ansatz of Calabi. A change of coordinates closely related to a Legendre transform converts the scalar curvature operator for such a metric into a very simple linear second-order operator; it is thereby possible to construct metrics of specified scalar curvature. I will describe these methods and their limitations. (Mostly joint work with Michael A. Singer, University of Edinburgh.)

LISA JEFFREY, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada
The Verlinde formula for moduli spaces of parabolic bundles

The moduli space $M(n,d)$ is an algebraic variety parametrizing those representations of the fundamental group of a punctured Riemann surface into the Lie group $\mathrm{SU}(n)$ for which a loop around the boundary is sent to a particular n-th root of unity multiplied by the identity matrix. If n and d are coprime it is in fact a Kaehler manifold. One may relax the constraint and study moduli spaces $M(a)$ parametrizing those representations for which the loop around the boundary is sent to an element conjugate to a, if a is some element in $\mathrm{SU}(n)$, and these are also Kaehler manifolds for a suitable class of a.

The Verlinde formula calculates the dimension of the space of holomorphic sections of certain line bundles over the spaces $M(n,d)$ and $M(a)$: these dimensions are in a sense the dimensions of the quantizations of these spaces. We recall how a new proof of the Verlinde formula for $M(n,d)$, given in joint work with F. Kirwan (Ann. Math., 1998) may be obtained, and show how to modify this proof to obtain a proof of the variant of the Verlinde formula which applies to $M(a)$.

ASKOLD KHOVANSKY, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada
Algebraic geometry and geometry of convex polyhedra

Newton polyhedra provide a connection between Algebraic Geometry and Geometry of Convex Polyhedra. This connection is useful in both directions. On the one hand it gives visible and understandable answers on the numerous questions which came from Algebraic Geometry. On the other hand it suggests algebraic intuition in the Geometry of Convex polyhedra. The talk will contain a review of the subject including some new results.

DON O'SHEA, Mt. Holyoke College, South Hadley, Massachusetts 01075, USA
Limits of tangent spaces to real surfaces

There is a lovely theory, due to Henry, Le, Teissier and others, describing the space of limits of tangent spaces at singular points of complex surfaces. This space encodes much delicate geometric information about the surface in a neighbourhood of the singular point. Such a theory would be highly desirable over the reals, but the complex results do not specialize in a straightforward manner. In the talk, we describe the complex theory briefly, and discuss some recent work which has cleared up a number of open questions in the real case.

In particular, let $X, 0$ be a real surface in $\mathbb{R}^n, 0$. We investigate the tangent semicone C^+ to the surface (by which we mean the set of all vectors which can be obtained as a limit of a sequence $t_i x_i$ with $t_i > 0$ and $x_i \in X$ where the x_i tend to 0) and the Nash space K of the surface (the set of all planes which can be obtained as a limit of tangent planes to X at smooth points of x tending to 0).

We prove a structure theorem for K analogous to, but different in some interesting respects, than that over the complexes established by Lê. We show that there is a sharp (and, to us, unexpected) dichotomy between exceptional rays in C^+ which are tangent to the singular locus of X and those that aren't. In the latter case, we determine precisely when a ray lying in the singular part of C^+ must be exceptional, and show that the set of elements in K containing the exceptional ray cannot contain discrete elements—in fact, we can give a lower bound on the size of this set. This suggests a possible algorithm for determining when and where C^+ (and the geometric tangent cone $C = C^+ \cup -C^+$) fails to be algebraic and, more speculatively, an algorithm for computing C^+.

EVGENII SHUSTIN, Tel-Aviv University, Tel-Aviv, Israel
Asymptotically proper bounds in the geometry of equisingular families of curves

Given a smooth complex algebraic hypersurface and an ample linear system on it, we consider the family of irreducible curves in this linear system having isolated singular points of given topological or analytic types, and we ask if this family is nonempty, smooth, irreducible and has expected dimension. We present sufficient conditions for the above "good" properties of equisingular families, expressed as upper bounds to the sums of certain invariants of singularities, and we show that these bounds are asymptotically proper, *i.e.* if the sum of the same singularity invariants exceeds the upper bound multiplied by an absolute constant, then the equisingular family is empty, or not smooth, or reducible, or has a nonexpected dimension.

Our approach consists in reducing the problem to h^1-vanishing for the ideal sheaves of zero-dimensional schemes associated to singularities, and in applying various h^1-vanishing criteria which come from Riemann-Roch, Kodaira theorem, Bogomolov's theory of unstable rank two vector bundles on surfaces, Castelnuovo function theory.

(Joint work with G.-M. Greuel and C. Lossen.)

YOSEF YOMDIN, Weizmann Institute of Science, Rehovot, Israel
A tractable problem on the way (hopefully) to counting limit cycles of polynomial vector fields

We show that a certain infinitesimal version of the problem of bounding the number of limit cycles of a polynomial vector field on the plane leads to an interesting question, which can be formulated very simply:

Let $P(x)$ and $q(x)$ be two polynomials. Consider the function $H(x,y) = \int_0^x q(t)/\big(1 - yP(t)\big)$.

1. Find all x for which $H(x,y)$ is identically zero in y.

2. For any fixed x estimate the number of (real) zeroes of $H(x,y)$ in y.

These questions turn out to be closely related to several fields in Analysis and Algebra, in particular to the Generalized moments problem, to the algebra of polynomials under composition and to D-modules. We plan to present some of these relations, an answer to the questions 1 and 2 and to discuss some additional results and problems in the above direction.

Discrete Geometry / Géométrie discrète

R. Connelly, R. Erdahl, M. Senechal and W. Whiteley, Organizers

VICTOR ALEXANDROV, Sobolev Institute of Mathematics, Novosibirsk, 630090 Russia

Sufficient conditions for the extendability of an N-th order flex of polyhedra

A very long-standing (and still open) problem is to prove that a smooth compact surface in Euclidean 3-space is rigid. Classical attempts to solve this problem were based on the notion of an infinitesimal flex. Since a counterexample was constructed by R. Connelly in the class of polyhedral surfaces, particular attention was given to studying infinitesimal flexes of polyhedra.

We give a new approach to describing a high order flex of polyhedra. The main results provide some sufficient conditions under which an infinitesimal flex of a polyhedron can be extended to an authentic flex. We discuss also results of computer experiments demonstrating that one set of our sufficient conditions is fulfilled for the Bricard octahedra.

More precisely, let Q be a closed polyhedron with v vertexes, e edges and triangular faces. A special bilinear map $B\colon \mathbb{R}^{3v} \times \mathbb{R}^{3v} \to \mathbb{R}^e$ is constructed in such a way that the vector

$$X_0 + tX_1 + \cdots + t^k X_k \quad (X_j \in \mathbb{R}^{3v}, j = 0, 1, \ldots, k) \tag{1}$$

is the k-th order infinitesimal flex of the polyhedron $X_0 = Q$ if and only if the equation

$$\sum_{j=0}^{n} B(X_j, X_{n-j}) = 0$$

holds for each $1 \le n \le k$. Put $A_0 X = B(X, X_0) + B(X_0, X)$. Suppose an infinitesimal flex $X_0 + tX_1 + \cdots + t^n X_n$ of the polyhedron Q is given. To extend this flex into a higher order one, it is necessary to solve the linear equation

$$A_0 X_{n+1} = \sum_{j=1}^{n} B(X_j, X_{n+1-j}).$$

The following sufficient condition for algorithmic verification of the flexibility of polyhedra is proven:

Theorem. *Let a polyhedron $Q = X_0$ has an infinitesimal flex $X_0 + tX_1 + \cdots + t^n X_n$ and let $L_n \subset \mathbb{R}^{3v}$ be the linear span of the vectors $X_0, X_1, \ldots, X_n$. Suppose the equation $A_0 X = -B(X_i, X_j) - B(X_j, X_i)$ has a solution $X \in L_n$ for all $1 \le i \le n$, $1 \le j \le n$. Then Q is flexible.*

Computer experiments show that the conditions of the Theorem are fulfilled with $n = 5$ for Bricard's octahedron of the second type.

VALERY ALEXEEV, Department of Mathematics, University of Georgia, Athens, Georgia 30605 USA

Families of algebraic varieties associated with cell decompositions

Fix a finite set S of integral points in a lattice L and let Q be their convex hull. To any cell decomposition of Q with vertices in S we associate: 1) a family of projective algebraic varieties, and 2) a family of pairs of projective algebraic varieties together with Cartier divisors.

Similarly, we construct families of the above two types for any periodic cell decomposition with integral vertices.

The parameter spaces of families of the second type for various cell decompositions fit together into a moduli space, a proper algebraic variety itself. One application of this construction is a canonical geometric compactification of the moduli of principally polarized abelian varieties.

P. ATELA, Department of Mathematics, Smith College, North Hampton, Massachusetts 01063-0001, USA
Periodicity in geometric dynamical models in Phyllotaxis

We will discuss stable periodic configurations that arise in the analysis of a dynamical geometric model of phyllotaxis.

LYNN M. BATTEN, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada
Blocking sets and security

There is much literature on blocking sets in projective planes. The abundance of blocking sets of various types in this setting makes them amenable to use in security systems. We explain how this can be done, and describe the reliability issues involved.

MARGARET M. BAYER, University of Kansas, Lawrence, Kansas 66045-2142, USA
Eulerian partially ordered sets

The face lattices of convex polytopes belong to the class of Eulerian partially ordered sets. In every interval of these ranked posets, the number of elements of even rank equals the number of elements of odd rank. The flag vector of a ranked poset gives the numbers of chains for the various rank sets. This talk discusses the closed convex cone of flag vectors of Eulerian posets. (The linear span is determined by the generalized Dehn-Sommerville equations.) The approach is based on work of Billera and Hetyei on flag vectors of ranked posets and uses "half-Eulerian" partially ordered sets.

ANDRÁS BEZDEK, The Mathematical Institute of the Hungarian Academy of Science, Budapest, Hungary, and Department of Mathematics, Auburn University, Auburn, Alabama 36849-5310, USA
A Sylwester type theorem on circles

In spite of the specific title, generalizations of two combinatorial geometric problems will be disscussed. The classical result of Sylvester shows that n points in the plane, not all on a common line, there exists an ordinary line (a line through exactly two points). The similar question about unit circles will be disscussed. A problem of V. Boltyanski (proved by A. V. Bogomolnaya, A. V. Nazarov and S. Rukshin) claims that if n points in a convex n-sided polygon are given, then using the sides and the given points n side-point pairs can be formed such that the triangles determined by the pairs cover the convex polygon. We strengthen this result by proving that the assumption according to which the given points are inside of the polygon is redundant. We also give a bound for the number of disjoint such triangles.

KAROLY BEZDEK, Department of Geometry, Eotvos University, H-1088 Budapest, Hungary and Department of Mathematics, Cornell University Ithaca, New York 14853-7901, USA

On a stronger form of Rogers' lemma and the minimum surface area of Voronoi cells in unit ball packings

Rogers' lemma is an essential tool for estimating the density of unit ball packings in Euclidean space. Also, it motivates several other techniques of the classical theory of packing. In this talk we prove a strengthening of Rogers' lemma and apply it to estimate the minimum surface area of Voronoi cells in unit ball packings. We prove a lower bound for the surface area of Voronoi cells of unit ball packings in d-dimensional Euclidean space. This bound is sharp for $d = 2$ and implies Rogers' upper bound for the density of unit ball packings in d-space for all $d > 1$. Finally, we strengthen these results for $d = 3$.

T. BISZTRICZKY, Department of Mathematics, University of Calgary, Calgary, Alberta T2N 2N4, Canada

A Signature Theorem for uniform oriented matroids

Let M be an uniform oriented matroid over a set E of n elements $e(1), e(2), \ldots, e(n)$ endowed with the linear order $e(1) < e(2) < \cdots < e(n)$. We say that $e(i)$ and $e(i+1)$ are successive elements of E. The Signature Lemma of Cordovil and Duchet states that if for any circuit C, (*) successive elements of E that are in C have alternating signs in C, then M is alternating with respect to the linear order. In the Signature Theorem, the condition (*) is weakened.

JIN-YI CAI, Department of Computer Science, University of Buffalo, Buffalo, New York 14260, USA

A new transference theorem in geometry of numbers with applications to Ajtai's connection factor

Geometry of Numbers was christened by Minkowski as a synthesis of geometric tools for number theoretic problems such as Diophantine approximations and quadratic forms.

Not only are they fascinating, lattice problems may provide a new class of public-key cryptographic protocols usable in any communication network, such as the internet and the World-Wide-Web. We will discuss a recent breakthrough of Ajtai on a connection of average-case and the worst-case complexity of lattice problems, and its implication for the design of provably secure public-key cryptography.

We prove a new transference theorem in the geometry of numbers, giving optimal bounds relating the successive minima of a lattice with the minimal length of generating vectors of its dual, improving previously the best transference theorem of this type due to Banaszczyk. We also prove a stronger bound for the special class of lattices possessing n^{ϵ}-unique shortest lattice vectors which play an important role in the Ajtai-Dwork Crypto-system.

The theorems imply consequent improvement of the Ajtai connection factors in the connection of average-case to worst-case complexity of the shortest lattice vector problem.

ROBERT CONNELLY, Cornell University, Ithaca, New York 14853, USA

Holes in a membrane: tension percolation

Consider a convex polygonal planar membrane made of inextendable material in three-space clamped on its boundary. Suppose a finite number of interior disjoint convex holes are created in the membrane. Depending on the configuration of holes, tension within the membrane can vanish on a region larger than just what is covered by the holes. We propose an algorithm that can calculate exactly those regions that must have no tension. This can be applied to the following problems.

1. Consider a random process that creates holes in a given convex polygon in the plane. At what point does the unstressed portion occupy most of the region?

2. Consider a finite collection of disjoint convex subsets of the plane. We find a tiling of the plane by convex sets, coming from the projection of a convex polyhedron in three-space, such that there is at least one, but a minimal number of subsets in each tile.

3. Start with a triangular lattice grid in the plane. Remove each edge with probability p. What is the critical value of p, where the lattice fails to support a positive stress in any edge?

This is joint work with Joe Mitchell and Konstatin Rybnikov.

H. S. MACDONALD COXETER, Mathematics Department, University of Toronto, Toronto, Ontario M5S 3G3, Canada

Whence does an ellipse look like a circle?

In Euclidean 3-space, a system of confocal quadric contains two degenerate members: a *focal ellipse* and a *focal hyperbola*, lying in perpendicular planes in such a way that the foci of each coincide with the vertices of the other. George Salmon discovered that, when viewed from any point on the focal hyperbola, the focal ellipse looks like a circle! It seems worth while, quite apart from the consideration of quadrics, to give an elementary solution to the problem of finding the locus of viewpoints from which a given ellipse looks like a circle (and later, of finding the two viewpoints from which every plane section of a prolate spheroid looks like a circle).

BALÁZS CSIKÓS, Eötvös University, Budapest, Rákóczi út 5, H-1088 Hungary

Some results around the Hadwiger-Kneser-Poulsen conjecture

In 1954–56 Poulsen, Kneser and Hadwiger formulated the following conjecture: *Let $B_1, \ldots, B_N$ and $B'_1, \ldots, B'_N$ be congruent balls in the Euclidean n-space with centers $p_1, \ldots, p_N$ and $p'_1, \ldots, p'_N$ respectively. If $p_i p_j \geq p'_i p'_j$ for all $1 \leq i < j \leq N$, then the volume of $\bigcup_{i=1}^N B_i$ is not less than that of $\bigcup_{i=1}^N B'_i$.* Although the conjecture is still open, even in the planar case, it can be proved on the additional condition that one can move the points p_i to the points p'_i continuously in such a way that the distances between the points decrease during the motion. We shall discuss generalizations of this theorem for balls in the spherical and hyperbolic space and also for domains obtained from balls by means of the operations $\cup$ and $\cap$. The proofs are based on some formulae for the variation of the volume and a suitable modification of the Dirichlet-Voronoi decomposition. Some applications will also be presented.

LUDWIG DANZER AND GERRIT VAN OPHUYSEN, Universität Dortmund, Facherbereich Mathematik, Lehrstuhl II, 44221 Dortmund, Germany

A species of planar triangular tilings with inflation factor $\sqrt{-\tau}$

Consider the set $\mathcal{F} = \{A, X\}$ of two right triangles in $\mathbb{E}^2$ with sides having squared lengths 1, τ, $1+\tau$ and τ, τ^2, $\tau+\tau^2$ respectively. These can be glued together (in only one way) to form a third right triangle $X \dot{\cup} A$ with sides of squared length $1+\tau$, $\tau+\tau^2$, $(q+\tau)^2$. We then consider the inflation rule $\mathrm{infl}(A) := X$, $\mathrm{infl}(X) := X \,\dot\cup\, A$. Interpreting $\mathbb{E}^2$ as $\mathbb{C}$ the inflation factor η becomes $i\sqrt{\tau}$ which is a complex Pisot-number. The species $\mathcal{S}(\mathcal{F}, \mathrm{infl})$ of all global $\mathcal{F}$-tilings created by infl has a unique deflation ("infl^{-1}") and hence is aperiodic. The set of all vertices can be shown to be a "model set" (Robert Moody), so the Fourier-transformation of the autocorrelation function is "pure point" with the Bragg-peaks located on the $\mathbb{Z}$-module $\frac{3-\tau}{5}\left\langle \begin{pmatrix} \sqrt{\tau} \\ 0 \end{pmatrix} \begin{pmatrix} \sqrt{\tau}^{-1} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ \tau \end{pmatrix} \right\rangle_Z$.

With $\mathcal{F}_c := \{A, B, C, X, Y, Z\}$, where B and C are congruent to A, while Y and Z are congruent to X, but all differently coloured, and

$$\begin{aligned} \mathrm{infl}_c(A) &:= X & \mathrm{infl}_c(X) &:= Y \,\dot\cup\, A \\ \mathrm{infl}_c(B) &:= Y & \mathrm{infl}_c(Y) &:= X \,\dot\cup\, C \\ \mathrm{infl}_c(C) &:= Z & \mathrm{infl}_c(Z) &:= Z \,\dot\cup\, B \end{aligned}$$

we receive the species $\mathcal{S}(\mathcal{F}_c, \mathrm{infl}_c)$ of coloured tilings. In contrast to $\mathcal{S}(\mathcal{F}, \mathrm{infl})$ the coloured species can also be defined by a perfect local matching rule. In fact the 42 coloured vertex- stars may serve as an atlas.

ROBERT DAWSON, Department of Mathematics and Computing Science, Saint Mary's University, Halifax, Nova Scotia B3H 3C3, Canada
What shape is a loaded die?

In this talk, I will describe work done by Wendy Finbow and myself on the geometry of regular polytopes. We call a polytope *loadable* if it contains a point P that is above only one facet F; if such a polytope is constructed with nonuniform density so that its center of gravity is at P, and is stood on a horizontal hyperplane, it will topple from any other facet onto F. We classify the loadable regular polytopes, and illustrate, as far as possible, where the center of gravity must go to accomplish the loading.

MICHEL DEZA AND MIKHAIL SHTOGRIN, DMI, Ecole Normale Superieure, 75005 Paris, France
Embedding of regular tilings and star-honeycombs

We review the regular tilings of d-sphere, Euclidean d-space, hyperbolic d-space and Coxeter's regular hyperbolic honeycombs (with infinite or star-shaped cells or vertex figures) with respect of possible embedding, isometric up to a scale, of their skeletons into a m-cube or m-dimensional cubic lattice.

The last remaining 2-dimensional case is decided: for any odd $m \geq 7$, star-honeycombs $m\frac{m}{2}$ are embeddable while $\frac{m}{2}m$ are not (unique case of non-embedding for dimension 2). As a spherical analogue of those honeycombs, we enumerate 36 Riemann surfaces representing all nine regular polyhedra on the sphere. Also non-embeddability of all remaining star-honeycombs (on 3-sphere and hyperbolic 4-space) is proved.

Finally, all cases of embedding for dimension $d > 2$ are identified. Besides hyper-simplices and hyper-octahedra, they are exactly those with bipartite skeleton: hyper-cubes, cubic lattices and 8, 2, 1 tilings of hyperbolic 3-, 4-, 5-space (only two, 435 and 4335, of those 11 are compact).

absRobert Erdahl Queen's University, Kingston, Ontario K7L 3N6, Canada Voronoi's hypothesis on perfect domains

It was implicit in Voronoi's famous last two geometrical memoirs that he felt the partition of the cone of metrical forms into lattice type domains is a refinement of the partition into perfect domains. This property of perfect domains is now referred to as Voronoi's Hypothesis. The first of these memoirs was on perfect forms and their associated domains, and in the second he introduced his famous theory of lattice types; so this hypothesis links these last two memoirs. Voronoi's hypothesis has played an important role in the programs to classify higher dimensional lattices that followed. More precisely, this hypothesis played a decisive role in the fundamental work on classification of Delaunay, and more recently of Ryshkov and Baranovski. Ryshkov and Baranovsky were able to show that Voronoi's hypothesis holds up to dimension 5. I will show that this convenient relationship does not hold in general by demonstrating that the L-type partition in 6 dimensions is not a refinement of the perfect partition. This work is joint with Konstantin Rybnikov.

FERENC FODOR, Department of Mathematics, Auburn University, Auburn, Alabama 36849-5310, USA
Large polygons in convex sets and polygons with large perimeter

In this talk we investigate the problem that how large convex n-gons can be inscribed in a convex disc $\mathcal{K}$ in its Minkowski metric. We show that in any convex disc we can always inscribe a triangle of minimal side length $2/3$, and in a centrally symmetric disc a triangle with sides not shorter than $0.773\ldots$. We also give a short construction for an inscribed triangle whose sides are at least $3/4$ in the Minkowski metric induced by $\mathcal{K}$.

We also consider the question of finding the maximum perimeter p_n of a convex n-gon of unit diameter. We prove that the diameter graph of such an extremal polygon must be connected. Therefore if $n = 2^k$ no regular polygon is extremal. We shall also present applications of these facts in connection with finite circle packing problems and diameter minimalization of finite point sets with mutual distances at least one.

Some of the above results are joint works with A. Bezdek and I. Talata.

DEBORAH S. FRANZBLAU, Department of Mathematics, CUNY/College of Staten Island, Staten Island, New York 10314, USA
Generic rigidity of molecular graphs

A hard open problem is to find a purely combinatorial algorithm to determine whether a graph, representing a structure made of rigid bars and flexible joints, is generically rigid in 3 dimensions. A *molecular graph* or *square* of a graph, in which an edge is added between each pair of neigboring edges, models an atom-bond network with constraints on bond angles as well as bond lengths, which is a special case important in materials science, especially for the study of glasses. In this talk I will discuss the computation of generic degrees of freedom, the standard measure of rigidity, of molecular graphs.

CHRISTOPHER GOLD, Chair in Geomatics, Laval University, Quebec City, Quebec G1K 7P4, Canada
Voronoi methods in geomatics—the importance of the spatial model

Geomatics covers surveying, remote sensing, geographic information systems (GIS) and other topics concerning the capture and management of spatial information. Thus the restricted (discrete) model of space used in the computer is of central importance. While known for many years, the simple Voronoi diagram is only now being recognized as a central organizing concept for many types of data. This has been helped by the discipline of Computational Geometry, and is now being recognized within Geomatics. This talk will focus on applications based on simple concepts, and show how the Voronoi model is ideally suited to both algorithm development and geographic data input, management and analysis.

The plenary address will survey some of the classical examples of these moduli spaces and their structures. The special session talk will describe some of the more technical details of the proof.

CHAIM GOODMAN-STRAUSS, Department of Mathematics, University of Arkansas, Fayetteville, Arkansas 72701, USA
Addressing and substitution tilings

We will discuss recent work concerning addressing in substitution tilings. Such addresses allow control over a variety of structures in such tilings, including adjancies, the combinatorics and the geometry itself of the tiling. In essence, there is a correspondence between the combinatorical structure of a substitution species and a automatically defined quotient on a regular language viewed as a topological Cantor set.

TIMOTHY F. HAVEL, Biological Chemistry and Molecular Pharmacology, Harvard Medical School, Boston, Massachusetts 02115, USA
The role of tensegrity in distance geometry

Distance geometry is an invariant-theoretic approach to Euclidean (and classical non-Euclidean) geometry, which was developed in large part by A. Cayley, K. Menger, I. J. Schoenberg, J. J. Seidel, and L. M. Blumenthal. This approach has proved useful in mathematical studies of the kinematics of mechanisms, and in numerical studies of the conformations of molecules. Tensegrity frameworks are an architectural novelty introduced by K. Snelson and popularized by B. Fuller, whose constructions and mathematical properties have been extensively studied by R. Connelly and colleagues. A key result has been an algebraic criterion which can identify tensegrity frameworks that are globally rigid, meaning that all realizations of the framework are congruent. This talk will describe the insights that Connelly's results have brought to the applications of distance geometry to molecular conformation, and conversely, some new insights which the perspective of distance geometry brings to the theory of tensegrity frameworks.

DONALD JACOBS, Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-1116, USA
Graph rigidity: applications to material science and proteins

The mechanical stability of network glasses and proteins can be effectively studied by modeling the microstructure as a generic bar-joint framework. Graph rigidity for bar-joint frameworks in two dimensions is completely characterized by Laman's theorem [1]. Recently, an efficient algorithm has been constructed to study generic rigidity percolation in two dimensions [2]. The desire to construct a similar algorithm in three dimensions is impeded by the lack of an analogous theorem. It has been proposed [3] that an all subgraph constraint counting characterization of generic rigidity is recovered in three-dimensional bar-joint networks having no implied hinge joints. Based on this proposition, an efficient combinatorial algorithm has been constructed for bond-bending networks, which have no implied-hinge joints. Complete agreement is found with exact calculations involving diagonalization of dynamical matrices, for systems up to 1000 degrees of freedom.

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GÁBOR KERTÉSZ, Department of Geometry, Eötvös Loránd University, H-1088 Budapest, Hungary
The Dido problem on planes of constant curvature

What is the tract of land of maximum area that can be fenced by a given collection of fence segments? The segments should be the sides of a chord polygon. The proof is simple if the segments can not cross. However the proof of the general case was unknown for decades. Fifteen years ago I found a proof. Almost the same considerations show that this so called Dido type theorem for segments holds even in the hyperbolic plane.

WŁODZIMIERZ KUPERBURG, Department of Mathematics, Auburn University, Auburn, Alabama 36849-5310, USA
Covering the cube with equal balls

Define $c^d(k)$, the *covering radius*, as the minimum radius of k congruent balls that can cover the unit d-dimensional cube. (The packing radius is defined similarly.) In general, the problem of determining the values of $c^d(k)$ as well as the corresponding packing problem seem to be extremely difficult even in dimension 2. On the packing problem, several results have been obtained in dimension 2 by various authors, and a few results in dimension 3, mainly by J. Schaer. We discuss the covering problem and we determine $c^d(2)$, $c^3(3)$, $c^3(4)$, $c^3(8)$, $c^4(4)$, and $c^4(16)$ along with the optimal configurations of balls that produce them. Also, we state conjectures on the remaining values of $c^3(k)$ and their ball configurations for $k \leq 12$.

BARRY MONSON, Department of Mathematics and Statistics, University of New Brunswick, Fredericton, New Brunswick E3B 5A3, Canada
Realizations of regular abstract polytopes

A *regular abstract polytope* $\mathcal{P}$ is a poset having the 'essential' structural features of the face lattice of a regular convex polytope, including transitivity of $\mathrm{Aut}(\mathcal{P})$ on flags. Other examples include regular tessellations and honeycombs, star-polyhedra and many less familiar beasts. Since $\mathcal{P}$ need not actually be a lattice or have a particularly nice geometric realization, it is interesting to describe all *realizations* for $\mathcal{P}$. The basic theory of realizations was developed by McMullen (1989), who used geometric methods to describe a crucial family of real representations for $\mathrm{Aut}(\mathcal{P})$.

After an explicit, but brief, description of these objects, I will discuss work with Asia Ivić Weiss, in which we construct families of regular polytopes parametrized by certain rings of algebraic integers. Typically, our understanding of the realization cones for these polytopes is very limited.

KONSTANTIN RYBNIKOV, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6, Canada
On traces of d-stresses in the skeletons of lower dimensions of homology manifolds

We show how d-stresses on piecewise-linear realizations of d-manifolds in $\mathbb{R}^d$ induce k-stresses, $0 \leq k \leq d-1$, and discuss possible connections between the constructed polynomial mappings and

g-vectors of manifolds. This is a joint work with R. Erdahl and S. Ryshkov.

IDZHAD KH. SABITOV, Faculty of Mechanics and Mathematics, Moscow State University, 119899 Moscow, Russia
Solution of polyhedra

By analogy with the chapter of elementary mathematics named "the solution of triangles" we can propose an idea for finding all metric characteristics of a polyhedron based on the knowledge of its metric and combinatorial structure. The essential moment of the proposed approach is the generalized Heron's formule for volumes of isometric polyhedra established in [1]. Namely we affirm that the volumes of all isometric polyhedra with a fixed combinatorial simplicial structure K and given lengths (l) of its edges are roots of a polynomial equation

$$Q(V) = V^{2N} + a_1(l)V^{2N-2} + \cdots + a_N(l) = 0,$$

whose coefficients are, in one's turn, polynomials in edges' lengths with some numerical coefficients depending only on the combinatorial structure K. For the construction of a such equation one can indicate some corresponding algorithm. It turns out moreover that for the polyhedra in general position we can find only a finite number of values of its diagonals so we have a finite algorithm for the construction of isometric polyhedra. Some simple cases of this algorihm are realized on the computer.

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PETER SCHMITT, Institut für Mathematik, Universität Wien, A-1090 Wien, Austria
The versatility of (small) sets of prototiles

A set of prototiles (which are usually assumed to be closed topological disks or balls) may or may not admit a tiling of the plane (or of n-space). The set of all (distinct) tilings admitted represents the *versatility* of a set of prototiles. It may be large or small, and the tilings may have quite different properties. The talk provides an survey of the versatility of small sets, with emphasis on periodicity properties, and describes some general constructions.

ISTVAN TALATA, Department of Mathematics, Auburn University, Auburn, Alabama 36849-5310, USA
On translative coverings of a convex body with its homothetic copies of given total volume

Let K be a d-dimensional convex body. Denote by $h(K)$ the minimum number of smaller homothetic copies of K which are needed to cover K. Furthermore, denote by $hv(K)$ the smallest real number with the property that every sequence of positive homothetic copies of K with total volume at least $hv(K)\mathrm{vol}(K)$ permints a translative covering of K. It is clear that $h(K) \leq hv(K)+1$.

It is proved by Rogers [1957] that for any d-dimensional convex body K there exists a covering of R^d with translates of K with density at most $\delta_d = d\log d + d\log\log d + 5d$. As already Rogers observed in 1967, this result implies that $h(K) \leq 2^d\delta_d$ for centrally symmetric convex bodies. Similarly, $h(K) \leq 4^d\delta_d$ was proved for arbitrary convex bodies.

In this talk we show that the method used by Rogers can be extended for homothetic copies of K with different coefficients. This way we can improve on the upper bound $hv(K) \leq (d+1)^d - 1$ of

Januszewski [1998] proving $hv(K) \leq 2^{d+o(d)}$ for centrally symmetric convex bodies, and $hv(K) \leq 4^{d+o(d)}$ for arbitrary d-dimensional convex bodies.

ANKE WALZ, Cornell University, Ithaca, New York 14853, USA
The Bellows conjecture in dimension four

I will talk about the following generalized version of the Bellows Conjecture: *"If S_t is a flex of an orientable singular cycle in Euclidean 4-space, then the volume of S_t is constant."* A *flex* is a continuous motion of a configuration of vertices that fixes the lengths of the edges of all triangles of the cycle.

I. Sabitov first proved this result for dimension $d = 3$ by using resultants to show that the volume of a polyhedron in 3-space is (up to a constant factor) integral over the squares of the edgelengths of the polyhedron. R. Connelly and A. Walz modified his proof, looking at places instead of resultants. I will show that this approach can be generalized to dimension 4. This provides a proof of the Bellows Conjecture in dimension 4.

ASIA WEISS, York University, Toronto, Ontario M3J 1P3, Canada
To be announced

WALTER WHITELEY, Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada
Constraining a spherical polyhedron with dihedral angles

The theorems of Cauchy, Dehn and Alexandrov describe which distance constraints on edges and face diagonals of spherical polyhedra are independent and which are sufficient for (local) rigidity of convex polyhedra. We consider alternate constraints: the face-vertex incidences and the dihedral angles along edges. Specific results include:

(a) the incidences alone are independent, and we then have at most $\min(|E| - 1, 2|F| - 3)$ additional independent dihedral angles;

(b) dependencies of dihedral angles (or self-stresses of the corresponding spherical framework on the Gaussian sphere) are isomorphic to parallel drawings of the polyhedron with a fixed point (isogonal maps preserving all angles);

(c) there are examples of additional dependencies which result in (infinitesimal) deformations of polyhedron preserving all incidences and dihedral angles—but not all face angles.

We *conjecture* that, for almost all proper realizations of any combinatorial type of spherical polyhedron with all dihedral angles fixed, there are only isogonal deformations.

Extremal Combinatorics / Combinatoire extrémale

D. deCaen, Organizer

RICHARD ANSTEE*, RON FERGUSON AND ATTILA SALI, Department of Mathematics, University of British Columbia, Vancouver, British Columbia V6T 1Z2, Canada; Department of Mathematics, University of British Columbia, Vancouver, British Columbia V6T 1Z2, Canada and Mathematics Institute of Hungarian Academy of Sciences, Budapest, Hungary

Small forbidden configurations

An extremal set problem stated in matrix terms considers a $k \times l$ $(0,1)$-matrix F and an $m \times n$ $(0,1)$-matrix A which has no repeated columns. We assume A has no submatrix which is a row and column permutation of F and then seek bounds on n in terms of F, m. We wish to report progress in obtaining a number of new exact bounds for various $2 \times l$ F. The arguments often use graph theory in interesting ways. Turán's $m^2/4$ bound arises with a different twist. It seems possible to get exact bounds for all $2 \times l$ F. Progress for $3 \times l$ F has also been obtained but the triple systems are harder to analyze.

AART BLOKHUIS, RALPH FAUDREE*, ANDRÁS GYÁRFÁS AND MIKLÓS RUSZINKÓ, Department of Mathematics and Computer Science, Technical University of Eindhoven; Department of Mathematical Sciences, University of Memphis, Memphis, Tennessee 38152, USA; Computer and Automation Research Institute, Hungarian Academy of Sciences, Budapest, Hungary and Computer and Automation Research Institute, Hungarian Academy of Sciences, Budapest, Hungary

Anti-Ramsey colorings in several rounds

For positive integers $k \le n$ and t let $\chi^t(k,n)$ denote the minimum number of colors such that at least in one of the t colorings of edges of K_n all $\binom{k}{2}$ edges of every $K_k \subseteq K_n$ get different colors. Generalizing a result of Körner and Simonyi, it is shown in this paper that $\chi^t(3,n) = \Theta(n^{1/t})$. Also two-round colorings in cases $k > 3$ are investigated. Tight bounds for $\chi^2(k,n)$ for all values of k except for $k = 5$ are obtained. Conversely, let $t(k,n)$ denote the minimum number of colorings such that—having the same $\binom{k}{2}$ colors in each coloring—at least in one of the total t colorings of K_n all $\binom{k}{2}$ edges of every $K_k \subseteq K_n$ get different colors. It is also shown, that for $k = n/2$ $t(k,n)$ is exponentially large. Several related questions are investigated.

JASON BROWN, Department of Mathematics and Statistics, Dalhousie University, Halifax, Nova Scotia B3H 3J5, Canada

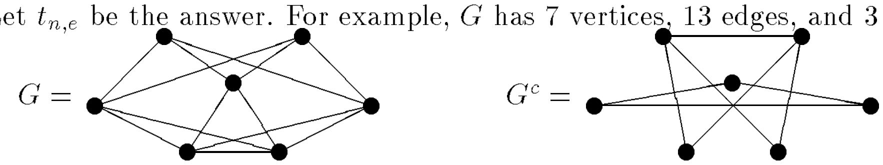
The inducibility of complete bipartite graphs

Given a fixed graph G, it is a natural extremal problem to ask: among all graphs of order n, which have the maximum number of induced subgraphs isomorphic to G? Furthermore, what is this maximum number? How can it be estimated? We examine here these problems for complete bipartite graphs. Surprisingly, the extremal graphs are not always complete bipartite graphs with equal (or nearly equal) parts. Some extensions to the multipartite case will also be noted.

DAVID FISHER, Department of Mathematics, University of Colorado at Denver, Denver, Colorado 80217-3364, USA

The minimum number of triangles in a graph

Given $0 \leq e \leq \binom{n}{2}$, what is the minimum number of triangles in a graph with n vertices and e edges? Let $t_{n,e}$ be the answer. For example, G has 7 vertices, 13 edges, and 3 triangles.



Since any graph with 7 vertices and 13 edges has at least 3 triangles, we have $t_{7,13} = 3$.

Trivially, $t_{n,e} = 0$ if $e \leq \frac{n^2}{4}$. McKay (1963) proved $t_{n,e} \geq \frac{(4e-n^2)e}{3n}$. Bollobás (1978) improved this by showing $t_{n,e}$ is at least the piecewise linear function with nodes $\left(\binom{k}{2}\left(\frac{n}{k}\right)^2, \binom{k}{3}\left(\frac{n}{k}\right)^3\right)$ for $k = 1, 2, \ldots$. Fisher (1989) found a better bound when $\frac{n^2}{4} \leq e \leq \frac{n^2}{3}$ by proving $t_{n,e} \geq \frac{9en-2n^3-2(n^2-3e)^{3/2}}{27}$. Nikiforov and Khadzhiivanov (1981) and Lovász and Simonovits (1983) independently found $t_{n,e} = (e - \lfloor \frac{n^2}{4} \rfloor)\lfloor \frac{n}{2} \rfloor$ for $\lfloor \frac{n^2}{4} \rfloor \leq e < \lfloor \frac{n^2}{4} \rfloor + \lfloor \frac{n}{2} \rfloor$.

I have a conjecture: *If G has n vertices, $e \geq \frac{n^2}{4}$ edges, and $t_{n,e}$ triangles, then G^c is not connected.* Barnhart and I verified this when $n \leq 14$. If true for all n, one can recursively find $t_{n,e}$. This would give analytic proofs of all results above plus a new bound similar to Fisher's when $e > \frac{n^2}{3}$.

JERROLD GRIGGS, Department of Mathematics, University of South Carolina, Columbia, South Carolina 29208, USA

Extremal graphs with bounded densities of small subgraphs

Let $\mathrm{Ex}(n, k, \mu)$ denote the maximum number of edges of an n-vertex graph in which every subgraph of k vertices has at most μ edges. Here we summarize some known results for the problem of determining $\mathrm{Ex}(n, k, \mu)$, present simple new proofs, and provide new estimates and extremal graphs. One of our main aims is to show how the classical Turán theory can be applied to such problems. The case $\mu = \binom{k}{2} - 1$ is the famous result of Turán. This is joint work with Miklós Simonovits and George Rubin Thomas.

PENNY HAXELL, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

Integer and fractional packings in dense graphs

Let H_0 be any fixed graph. For a graph G we define $\nu_{H_0}(G)$ to be the maximum size of a set of pairwise edge-disjoint copies of H_0 in G. We say a function ψ from the set of copies of H_0 in G to $[0, 1]$ is a *fractional H_0-packing* of G if $\sum_{H \ni e} \psi(H) \leq 1$ for every edge e of G. Then $\nu^*_{H_0}(G)$ is defined to be the maximum value of $\sum_{H \in \binom{G}{H_0}} \psi(H)$ over all fractional H_0-packings ψ of G. We show that $\nu^*_{H_0}(G) - \nu_{H_0}(G) = o(|V(G)|^2)$ for all graphs G. (Joint work with V. Rödl.)

FELIX LAZEBNIK, Department of Mathematical Sciences, University of Delaware, Newark, Delaware 19716, USA

On a class of algebraically defined graphs

Let F^n denote be the n-dimensional vector space over a field F. For $n \geq 2$ and each $i = 1, 2, \ldots, n-1$, let $f_i \colon F^{2i} \to F$ be a function of $2i$ variables. We consider a bipartite graph whose vertex partitions P and L are copies of F^n with $p = (p_1, p_2, \ldots, p_n) \in P$ and $l = (l_1, l_2, \ldots, l_n) \in L$ being joined by an edge if and only if the following $n-1$ equalities are satisfied:

$$\begin{aligned} l_2 + p_2 &= f_1(p_1, l_1) \\ l_3 + p_3 &= f_2(p_1, l_1, p_2, l_2) \\ &\vdots \\ l_n + p_n &= f_{n-1}(p_1, l_1, p_2, l_2, \ldots, p_{n-1}, l_{n-1}) \end{aligned}$$

For particular fields F and particular functions f_i's, the families of graphs defined this way (or slightly modified) possesses many remarkable properties. They are concerned with forbidden cycles, girth, graph homomorphism, eigenvalues, and edge-decompositions of complete graphs and complete bipartite graphs. In this talk we survey some known and some new results on such graphs based on the work of A. J. Woldar and the speaker.

LASZLO SZÉKELY, University of South Carolina, Columbia, South Carolina 29208-0001, USA
Some new Erdős-Ko-Rado type problems

I will survey some recent variants of the classic Erdős-Ko-Rado theorem: intersecting chains of sets, intersecting chains of subspaces, intersecting chains in posets, higher-order Erdős-Ko-Rado theorem, Erdős-Ko-Rado theorem for arithmetic progressions.

BING ZHOU, Trent University, Peterborough, Ontario, Canada
Extremal graphs related to star chromatic number and fractional chromatic number

We present results related to two generalizations of the chromatic number of a graph: the star chromatic number χ^* and fractional chromatic number χ_f.

In general we have the inequality $2 \leq \chi_f(G) \leq \chi^*(G) \leq \chi(G)$ for any graph G. We will study the graphs that have $\chi^*(G) = \chi(G)$ in relation with other properties of the graphs such as girth and planarity. Similar problems will be considered for the fractional chromatic number as well.

Number Theory / Théorie des nombres

R. Murty and N. Yui, Organizers

A. AKBARY, Department of Mathematics and Statistics, Concordia University, Montreal, Quebec H3G 1M8, Canada

On the distribution of the values of symmetric square L-functions in the half plane $\mathrm{Re}\,(s) > \frac{3}{2}$

Let $L_{\mathrm{sym}^2(f)}(s)$ be the symmetric square L-function associated to a newform of weight 2 and level N. We will derive an asymptotic formula for the average values of $L_{\mathrm{sym}^2(f)}(s)$ at a point s_0 in the half plane $\mathrm{Re}\,(s) > \frac{7}{4}$. Assuming the Riemann hypothesis for the Riemann zeta function, we are able to extend our result to the half plane $\mathrm{Re}\,(s) > \frac{3}{2}$.

HENRI DARMON, McGill University, Montreal, Quebec H3A 2K6, Canada

Modularity of hypergeometric abelian varieties

I will explain how Wiles result can be applied to prove the modularity of a large class of abelian varieties with real multiplication occuring as specialisations of "rigid families".

CHANTAL DAVID, Department of Mathematics and Statistics, Concordia University, Montreal, Quebec H3G 1M8, Canada

To be announced

JACEK W. FABRYKOWSKI, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

To be announced

G. FREI, Department of Mathematics and Statistics, University of Laval, Montreal, Quebec G1K 7P4, Canada

On the discovery of the reciprocity law by Artin

EYAL GOREN, Department of Mathematics, CICMA Concordia and McGill, McGill University, Montreal, Quebec H3A 2K6, Canada

Stratifications of moduli spaces and modular forms

We present recent results (some in collaboration with E. Bachmat and with F. Oort) on stratification of Hilbert and Drinfel'd modular varieties in finite characteristic, which is related to modular forms on these varieties. Special values of L-functions play an interesting role.

C. GREITHER, Département de mathématiques et statistique, Université de Laval, Montréal, Québec G1K 7P4, Canada

On Brumer's conjecture

Stickelberger showed around 1890 that a certain ideal I in the integral group ring $\mathbf{Z}G$ annihilates the class group of an abelian extension L with Galois group G over the field of rational numbers. The ideal I is constructed algebraically, but it has an interpretation by means of special values of L-functions as well. Stickelberger's proof is constructive in the sense that the required generators of principal ideals are written down as explicitly as one could ask for.

If now L is a complex abelian extension of a totally real field K, one can construct a Brumer-Stickelberger ideal I by reversing history, that is, by starting from special L-values. The Brumer conjecture then predicts annihilation of the class group $\mathrm{Cl}(K)$ by I (with some precautions); it is not proved yet in general. The talk will explain how to prove this conjecture for some classes of extensions L/K. The essential ingredients are the theory of Fitting ideals, Iwasawa theory, and the clever method of Wiles to bypass trivial zeros. Nobody knows how to get the required generators explicitly; the proof is highly nonconstructive.

At the end of the talk, some numerical and heuristical remarks concerning the $\mathbf{Z}G$-structure of class groups will be presented, in the spirit of the Cohen-Lenstra heuristics.

JAMES HUARD, Department of Mathematics, Canisius College, Buffalo, New York 14208, USA

An arithmetic reciprocity relation of Liouville type and applications

Let n be a positive integer and let $f\colon \mathbf{Z} \to \mathbf{C}$ be an even function. Liouville gave the identity

$$\sum_{aA+bB=n} \big(f(a-b)-f(a+b)\big) = f(0)\big(\sigma(n)-\tau(n)\big) + \sum_{d|n} f(d) + 2n\sum_{d|n}\frac{f(d)}{d} - \sum_{d|n} df(d) - 2\sum_{d|n}\sum_{1\le v\le d} f(v),$$

where the sum on the left hand side is taken over all positive integers a, A, b, B satisfying $aA+bB = n$. Some generalizations of this identity will be presented and applications will be made to the determination of such sums as

$$\sum_{m=1}^{n-1} \sigma_3(m)\sigma_3(n-m)$$

and to the representation of n by certain positive quaternary quadratic forms. The proofs of these results require elementary methods. This is a joint work with Z. Ou, B. K. Spearman, and K. S. Williams.

HERSHEY KISILEVSKY, Concordia University, Montreal, Quebec H3G 1M8, Canada

Rank of $E(K)$ for cyclic cubic extensions $K/\mathbf{Q}$

For an elliptic curve, E, defined over the rational field, we consider the rank of $E(K)$ for cyclic cubic extensions $K/\mathbf{Q}$.

MANFRED KOLSTER, Department of Mathematics, McMaster University, Hamilton, Ontario L8S 4K1, Canada

Higher relative class number formulas

Let F/F^+ be a CM extension of number fields, and let χ denote the non-trivial character of the Galois group of F/F^+. The classical *relative class number formula* gives the following relation between the value of the L-function of χ at 0 and the relative class number h^-:

$$L_{F^+}(\chi, 0) = \frac{2^{r_1}}{Q} \cdot \frac{h^-}{w(F)},$$

where $r_1 = [F^+ : \mathbb{Q}]$, Q is the so-called Q-index, and $w(F)$ is the number of roots of unity in F. The purpose of the talk is to describe analogs of this formula relating the values $L_{F^+}(\chi, 1-n)$ for $n \geq 3$ an odd integer to the orders of relative motivic cohomology groups and K-groups.

ARNE LEDET, Department of Mathematics, Queen's University, Kingston, Ontario K7L 3N6, Canada
Some small 2-groups as Galois groups

In 1936, Witt gave a criterion for embedding a V_4-extension into a Q_8-extension in terms of an equivalence of quadratic forms. Using this result, it is possible to give similar criteria for solving other embedding problems, starting with embedding a D_4-extension into a QD_8-extension, where D_4 and QD_8 are the dihedral and quasi-dihedral groups of order 8 and 16, resp. This again permits a complete description of extensions with certain 2-groups of order 16 as Galois groups.

C. LEVESQUE, Département de mathématiques et statistique, Université de Laval, Montréal, Québec G1K 7P4, Canada
Explicit solutions of a family of Thue diophantine equations

Looking for units of certain number fields built from modular coverings $X_0(m)$, H. Darmon obtained the family of polynomials

$$F_a(X,Y) = X^5 + 2X^4Y + (a+3)X^3Y^2 + (2a+3)X^2Y^3 + (a+1)XY^4 - Y^5.$$

For a sufficiently large, O. Kihel exhibited a fundamental system of units of the field $\mathbf{Q}(\omega)$ where ω is a root of $F_a(X,1) = 0$. In this lecture we will show that for a sufficiently large, the family of Thue diophantine equations of degree 5 given by

$$F_a(X,Y) = \pm 1$$

has only the six trivial solutions $(x,y) = (0,1), (0,-1), (1,0), (-1,0), (1,-1), (-1,1)$. The associated Siegel equation leads firstly to a linear form in four logarithms, but the main ingredient and the feature of the proof is to write it as a linear form in two logarithms.

KUMAR MURTY, Mathematics Department, University of Toronto, Toronto, Ontario M5S 3G3, Canada
Zeros of Dedekind zeta functions in towers of fields

We shall discuss some recent results about the non-existence of zeros of Dedekind zeta functions near $s = 1$ as we vary over a tower of fields.

W. GEORG NOWAK, Universität für Bodenkultur, A-1180 Vienna, Austria
Large convex domains sometimes contain more lattice points than we would expect

Let as usual $r(n)$ denote the number of ways to write $n \in \mathbf{N}$ as a sum of two squares. Then the quantity

$$P(t) = \sum_{0 \le n \le t} r(n) - \pi t$$

(the "lattice rest" of an origin-centered circle of radius $\sqrt{t}$) is well-known to satisfy

$$\int_0^X \left(P(t)\right)^2 dt \sim C X^{3/2} \tag{1}$$

(Cramér, Landau),

$$\liminf_{t \to \infty} \frac{P(t)}{(t \log t)^{1/4}} < 0 \tag{2}$$

(Hardy), and

$$\limsup_{t \to \infty} \frac{P(t)}{t^{1/4} \exp\bigl(c (\log \log t)^{1/4} (\log \log t)^{-3/4}\bigr)} > 0 \tag{3}$$

(Corrádi & Kátai). Compared to (2), (3) is not only weaker but also incapable of generalisation to more general domains, its proof being based on rather special "arithmetic" arguments.

The present talk addresses the corresponding problem for cubes: Let

$$r_3(n) = \#\{(u,v) \in \mathbf{Z}^2 : |u|^3 + |v|^3 = n\},$$

and denote by $P_3(t)$ the error term in the asymptotic formula for $\sum_{0 \le n \le t^{3/2}} r_3(n)$. Combining classic analytic number theory with some profound algebra and a very recent deep result of Heath-Brown [1], the authors [2] where able to show that

$$\limsup_{t \to \infty} \frac{P_3(t)}{(t \log \log t)^{1/4}} > 0.$$

This is based on the fact that the set

$$\{(m^{3/2} + n^{3/2})^{2/3} : (m,n) \in \mathbf{N}^2, \gcd(m,n) = 1\}$$

contains a rather "large" set of numbers which are linearly independent over the rationals.

The corresponding general problem for $r_k(n)$, $k > 3$, remains open at the present state-of-art.

References

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YANNIS PETRIDIS, McGill University, Montreal, Quebec H3A 2K6, Canada

Zeros of the Riemann zeta function and central values of L-series of holomorphic cusp forms

We examine the stability of the zeros of the Riemann zeta function, which are twice the scattering poles of $\mathrm{SL}(2,Z)$, in relation to the central value of the L-series of holomorphic cusp forms of weight 2 for the congruence subgroups $\Gamma_0(q)$, q prime. We work with perturbations in characters varieties of $\Gamma_0(q)$ and study the effects on the spectral and scattering theory of the Laplace operator.

GAEL RÉMOND, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada
Theta heights and Kodaira construction

We prove that in Kodaira construction a height on the base curve is described by the theta height of the jacobians of the fibres. We make use of theta heights as introduced by Bost and David who prove a comparison with Faltings' height. Therefore our result shows that a suitable effective version of Shafarevich conjecture (a bound for Faltings' height) implies an effective Mordell.

D. ROY, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada
Criteria of algebraic independence and approximation by hypersurfaces

Given a point θ in $\mathbb{C}^m$, a fundamental problem is how close one can approximate θ by a point of an algebraic variety of dimension d, defined over $\mathbb{Q}$, with degree $\leq D$ and logarithmic height $\leq T$. The problem has a different flavor whether, for a fixed d, one wants an estimate valid for a pair (D,T) or for infinitely many pairs (D_n,T_n) chosen from a given non-decreasing sequence of positive integers $(D_n)_{n\geq 1}$, and a given non-decreasing unbounded sequence of positive real numbers $(T_n)_{n\geq 1}$ with $T_n \geq D_n$ for each $n \geq 1$. In a joint work with Michel Laurent, we analyze the second type of problem when $d = m-1$. We show that, for infinitely many indices n, there exists a nonzero polynomial $P \in \mathbb{Z}[X_1,\ldots,X_m]$ of degree $\leq D_n$ whose coefficients have absolute value $\leq \exp(T_n)$, such that P admits at least one zero α in $\mathbb{C}^m$ with

$$\|\theta - \alpha\| \leq \exp\bigl(-c(m) D_{n-1}^m T_{n-1}\bigr)$$

for some positive constant $c(m)$ which depends only on m. This follows from a new criteria of algebraic independence with multiplicities. The object of the talk is to explain the link between the two results.

GARY WALSH, Department of Mathematics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada
Old and new results on quartic diophantine equations

During his illustrious career, Wilhelm Ljunggren proved many results on the solvability of Diophantine equations of the form $aX^4 - bY^2 = c$. In this talk we present some of Ljunggren's work, some problems remaining from his work, and recent progress on these outstanding problems. This is joint work with M. A. Bennett.

HUGH WILLIAMS, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada
Computer verification of the Ankeny-Artin-Chowla conjecture for all $p < 5.10^{10}$

Let p be a prime congruent to 1 modulo 4 and let t, u be rational integers such that $(t+\sqrt{p}u)/2$ is the fundamental unit of the real quadratic field $\mathbf{Q}(\sqrt{p})$. The Ankeny-Artin-Chowla conjecture (AAC conjecture) asserts that p will not divide u. This is equivalent to the assertion that p will not divide $B_{(p-1)/2}$, where B_n denotes the n-th Bernoulli number. Although first published in 1952, this conjecture still remains unproved today. Indeed, it appears to be most difficult to prove.

Even testing the conjecture can be quite challenging because of the size of the numbers t, u; for example, when $p = 40094470441$, then both t and u exceed 10^{330000}. In 1988 the AAC conjecture was verified by computer for all $p < 10^9$. In this paper we describe a new technique for testing the AAC conjecture and we provide some results of a computer run of the method for all primes p up to 5.10^{10}.

This is joint work with Alf van der Poorten and Herman te Riele.

KENNETH WILLIAMS, School of Mathematics and Statistics, Carleton University, Ottawa, Ontario K1S 5B6, Canada

Values of the Dedekind eta function at quadratic irrationalities

The Dedekind eta function $\eta(z)$ is defined by

$$\eta(z) = e^{\pi i z/12} \prod_{m=1}^{\infty} (1 - e^{2\pi i m z})$$

for $im(z) > 0$. Let d be the discriminant of an imaginary quadratic field. The value of $\left|\eta\left((b + \sqrt{d}/2a\right)\right|$ is determined for integers a, b, c satisfying

$$b^2 - 4ac = d, \quad a > 0, \ (a, b, c) = 1.$$

This is joint work with A. J. van der Poorten.

Operator Algebras / Algèbres d'opérateurs

J. Mingo, Organizer

BERNDT BRENKEN, Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta T2N 1N4, Canada
Endomorphisms of finite direct sums of I_∞ factors

The well known correspondence between $*$-representations of Cuntz algebras and unital $*$-endomorphisms of $\mathcal{B}(\mathcal{H})$ is extended to a correspondence between $*$-representations of Cuntz-Krieger algebras and unital $*$-endomorphisms of finite direct sums of I_∞ factors.

KENNETH R. DAVIDSON, Pure Mathematics Department, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
Principal bimodules of nest algebras

We classify the WOT-closed bimodules over a pair of nest algebras which are singly generated as algebraic and as norm-closed bimodules. The obstructions relate to the finite rank atoms. In particular, if both nest algebras have infinite multiplicity, then every WOT-closed bimodule is (algebraically) principal. Another important special case is the ideal of strictly upper triangular operators, which is always principal; and the generator is a sum of commutators. In general, every countably generated WOT-closed bimodule is singly generated, and we obtain explicit bounds on the number and norms of the terms in a factorization through the generator.

GEORGE ELLIOTT, Fields Institute, Toronto, Ontario, Canada
An abstract Brown-Douglas-Fillmore absorption theorem, II

The criterion of Elliott and Kucerovsky (see Part I) for an extension of a stable, nuclear, separable C^*-algebra by an arbitrary separable C^*-algebra to be absorbing—namely, that it be what is called "purely large"—can be verified directly in all known cases.

This is joint work with Dan Kucerovsky.

THIERRY GIORDANO, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada
Ergodic theory and dimension G-spaces

Let G be a discrete group. The real group algebra $A = \mathbb{R}G$ has a natural order structure given by the positive cone $A_+ = \{\sum r_g g\ ;\ r_g = 0 \text{ a.e. and } r_g \geq 0\}$ and is endowed with an order-preserving action of G (by right multiplication). If $n \in \mathbb{N}$, then A^n is a partially ordered vector space with the direct sum ordering and a G-space with the above G-action.

Definition. A G-dimension space H is a partially ordered vector space with an action of G (as a group of order automorphisms) that can be obtained as a direct limit

$$H = \varinjlim A^{n(i)} \xrightarrow{\phi_i} A^{n(i+1)}, \tag{1}$$

where ϕ_i is a positive A-module map (for the natural ordering on A).

Corresponding to the inductive limit in (1) is a matrix-valued random walk on G. The harmonic functions associated to this random walk are in a natural bijection with the states on H. A state γ corresponding to a bounded harmonic function is called bounded. It induces a pseudo-norm on H and allows us to associate to H the real L^1-space $L^1(X)$, the "completion" of H (as defined by Goodearl and Handelman). If for all $g \in G$, $g\gamma \leq N(g)\gamma$ ($N(g)$ depending on g), then G acts on $L^1(X)$.

D.E. Handelman and I have defined the notion of ergodicity and different generalizations of approximate transitivity for the action of G on (H, γ) which extends to $L^1(X)$ and its dual.

DAN KUCERVOSKY, Fields Institute, Toronto, Ontario, Canada

An abstract Brown-Douglas-Fillmore absorption theorem, I

A common generalization is given of all previous absorption theorems for C^*-algebra extensions. We consider extensions of a stable, nuclear, separable C^*-algebra by an arbitrary separable C^*-algebra, and in this setting, we give a simple intrinsic criterion for an extension to be absorbing. The criterion involves only the C^*-algebra of the extension, together with the canonical closed two-sided ideal. This is joint work with George Elliott.

MICHAEL LAMOUREUX, Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta T2N 1N4, Canada

Crossed product algebra constructions

By a number of examples, we demonstrate the relationships between C^*-crossed product algebras that arise from group actions, semigroup actions, and partial actions, and the corresponding nonselfadjoint crossed products. The connection between the corresponding ideal structures will be discussed.

ALEXANDRU NICA, Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

Some minimization problems for the free analogue of the Fisher information

We consider the free non-commutative analogue Φ^*, introduced by D. Voiculescu, of the concept of Fisher information for random variables. We determine the minimal possible value of $\Phi^*(a, a^*)$, if a is a non-commutative random variable subject to the constraint that the distribution of a^*a is prescribed. More generally, we obtain the minimal possible value of $\Phi^*(\{a_{ij}, a_{ij}^*\}_{i,j})$, if $\{a_{ij}\}_{1\leq i,j\leq d}$ is a family of non-commutative random variables such that the distribution of A^*A is prescribed, where A is the matrix $(a_{ij})_{i,j=1}^d$. The $d \times d$-generalization is obtained from the case $d = 1$ via a result of independent interest, concerning the minimal value of $\Phi^*(\{a_{ij}, a_{ij}^*\}_{i,j})$ when the matrix $A = (a_{ij})_{i,j=1}^d$ and its adjoint have a given joint distribution. (A version of this result describes the minimal value of $\Phi^*(\{b_{ij}\}_{ij})$ when the matrix $B = (b_{ij})_{i,j=1}^d$ is selfadjoint and has a given distribution.)

We then show how the minimization results obtained for Φ^* lead to maximization results concerning the free entropy χ^*, also defined by Voiculescu.

This is joint work with Dimitri Shlyakhtenko and Roland Speicher.

JOHN PHILLIPS, Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3P4, Canada
Spectral flow and index in bounded and unbounded θ-summable Fredholm modules: integral formulas

In joint work with Alan Carey, we study θ-summable Fredholm modules (H, D_0) for Banach $*$-algebras, A, and integral formulas for the pairing of (H, D_0) with $K_1(A)$. In particular, if (H, D_0) is θ-summable (in Connes' original sense that $\mathrm{Tr}(e^{-tD_0^2}) < \infty$ for all $t > 0$) then we prove that if u is a unitary in A^1 with $[u, D_0]$ bounded and $P = \chi_{[0,\infty)}(D_0)$, then

$$\text{index}\,(PuP) = \frac{1}{\sqrt{\pi}} \int_0^1 \mathrm{Tr}\Big(\frac{d}{dt}(D_t) e^{-D_t^2}\Big)\, dt$$

where $D_t = D_0 + t(uD_0u^* - D_0)$ is the straight line path from D_0 to uD_0u^*. This is the pairing of $[u]$ in $K_1(A)$ with $[D_0]$ in $K^1(A)$ and can also be interpreted as the spectral flow of the path $\{D_t\}$. A proof of this formula was first outlined by Ezra Getzler.

Our proof is quite different from the one indicated by Getzler. Our method is able to handle Connes' new notion of θ-summability (which we dub weak-θ-summability) *i.e.*, $\mathrm{Tr}(e^{-tD_0^2}) < \infty$ for *some* $t > 0$. We can also handle the various bounded notions of θ-summability (indeed, our method is based on this). Finally our method simultaneously deals with the II_∞-version of all these results.

Roughly speaking, our proofs involve three steps. First, given (H, D_0) we make a careful analytic study of the map $D \mapsto F_D := D(1 + D^2)^{-\frac{1}{2}}$ defined for D in $D_0 + \mathcal{B}(H)_{\mathrm{sa}}$. The pair (H, F_{D_0}) is a pre-Fredholm module for A which is θ-summable (in a restricted sense) and the F_D vary in a certain affine space of bounded self-adjoint operators, $F_{D_0} + \mathcal{L}_{\mathrm{sa}}$. We show that this map

$$D \mapsto F_D \colon D_0 + \mathcal{B}(H)_{\mathrm{sa}} \to F_{D_0} + \mathcal{L}_{\mathrm{sa}}$$

is suitably smooth. Second, we study the various notions of bounded θ-summability that arise in this context. In particular, one of our results shows that if (H, F_0) is a pre-Fredholm module which is θ-summable for A in the strongest sense, then for F in $F_0 + \mathcal{L}_{\mathrm{sa}}$ and X in $\mathcal{L}_{\mathrm{sa}}$ (the tangent space to $F_0 + \mathcal{L}_{\mathrm{sa}}$ at F), $\mathrm{Tr}(Xe^{-|1-F^2|^{-1}})$ is a closed (and exact) 1-form on the manifold, $F_0 + \mathcal{L}_{\mathrm{sa}}$. The idea to use these differential-geometric notions in this context goes back to I.M. Singer and was also used by Getzler in his proof. The point here is that now the integral

$$\int_0^1 \mathrm{Tr}\Big(\frac{d}{dt}(F_t) e^{-|1-F_t^2|^{-1}}\Big)\, dt$$

is seen to be independent of path in the space, $F_0 + \mathcal{L}_{\mathrm{sa}}$. Third, we can now reduce to the case of genuine (bounded, θ-summable) Fredholm modules (H, F_0) (*i.e.*, $F_0^2 = 1$), and paths $F_t = F_0 + t(uF_0u^* - F_0)$. In this setting, $P = 2F_0 - 1$ and the formula

$$\mathrm{Index}(PuP) = \frac{1}{C} \int_0^1 \mathrm{Tr}\Big(\frac{d}{dt}(F_t) e^{-|1-F_t^2|^{-1}}\Big)\, dt$$

is still *not* a triviality.

As mentioned above, our results are proved in such a way that the type II_∞ case is included at all stages.

JACK SPIELBERG, Department of Mathematics, Arizona State University, Tempe, Arizona 85287-1804, USA
A new look at C^*-algebras of infinite graphs

I will describe a new approach to the generalization of Cuntz-Krieger algebras from finite to infinite matrices. I will compare this with other methods.

SAM WALTERS, Department of Mathematics and Computer Science, University of Northern British Columbia, Prince George, British Columbia V2K 4A2, Canada
K-theory of non commutative spheres arising from the Fourier automorphism

It is shown that for a dense G_δ set of the real number θ (containing the rationals) there is an isomorphism $K_0(A_\theta \rtimes_\sigma \mathbb{Z}_4) \cong \mathbb{Z}^9$, where A_θ is the rotation C^*-algebra generated by unitaries U, V satisfying $VU = e^{2\pi i\theta}UV$ and σ is the Fourier automorphism given by $\sigma(U) = V$, $\sigma(V) = U^{-1}$. More precisely, a basis consisting of nine canonical modules is explicitly given. It is also shown that for a dense G_δ one has $K_1(A_\theta \rtimes_\sigma \mathbb{Z}_4) = 0$.

Probability Theory / Théorie des probabilités

M. Csörgő, Organizer

SIVA ATHREYA, The Fields Institute, 222 College Street, Fields Institute, Toronto, Ontario M5T 3J1 Canada
Existence of Positive Solutions Satisfying the Boundary Harnack Principle for a Semi-linear Dirichlet Problem

Boundary Harnack principle is a key tool in obtaining many results in classical potential theory. Suppose D is a smooth domain and u and v are two positive harmonic functions on D that vanish on a subset A of ∂D. The boundary Harnack principle says that u and v tend to zero at the same rate. Over the past three decades, there has been a lot of research on extending the principle to very general domains.

Another natural question is, does the boundary Harnack principle hold for solutions of elliptic partial differential equations other than $\triangle u = 0$? We shall investigate the above question in our talk. This was part of my Ph.D. thesis work with Professor K. Burdzy, at the University of Washington.

CLAUDE BELISLE, Département de mathématiques et statistique, Université de Laval, Montréal, Québec G1K 7P4, Canada
The Hit-and-run sampler

The *hit-and-run* sampler is a Markov Chain Monte Carlo method for simulating probability measures.

Let π be an absolutely continuous probability measure on $\mathbb{R}^d$. Let ν be a full dimensional probability measure on the surface S of the d-dimensional unit ball centered at the origin. Given a *current* point $X_n \in \mathbb{R}^d$, the hit-and-run sampler chooses a next point X_{n+1} according to the conditionalization of π on the line through X_n and $X_n + \Theta_{n+1}$. The *directions* $\Theta_1, \Theta_2, \Theta_3, \ldots$ are independent and identically distributed on S, with distribution ν. Under an appropriate irreducibility condition, the Markov chain $(X_n\ ;\ n \geq 0)$ converges in total variation towards the *target* distribution π. In this talk, I will discuss the convergence properties of this Markov chain. Related Markov Chain Monte Carlo methods, including the Gibbs sampler, will also be discussed.

DAVID BRILLINGER, Statistics Department, University of California, Berkeley, California 94720-3860, USA
Some aspects of the motion of particles described by stochastic differential equations

Consider a particle moving on the surface of the unit sphere in R^3 and heading towards a specific destination with a constant average speed, but subject to random deviations. The motion is modelled as a diffusion with drift restricted to the surface of the sphere. Expressions are set down for various characteristics of the process including expected travel time to a cap, the limiting distribution, the likelihood ratio and some estimates for parameters appearing in the model.

The cases of motion near the surface of the sphere and where explanatory variables are present are also considered.

MURRAY D. BURKE, Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta T2N 1N4, Canada
Model checking and estimation: A large sample approach

A brief survey of some models used in statistical analysis is given. Estimation and tests of fit considerations often lead to distribution-free or asymptotically distribution-free statistics. However, in many types of situations (*e.g.* multivariate, semiparametric models), the resulting processes depend on the underlying distribution function or family of distribution functions. Weighted bootstrap methods are considered as an omnibus way of approximating the limiting distribution of test statistics. Since percentage points of the resulting statistics do not need to be calculated explicitly, one can be flexible in the choice of statistic and achieve greater power against particular alternatives.

COLLEEN D. CUTLER, Department of Statistics and Actuarial Science, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
Scaling structures, chaos, and determinism in time series

The influence of ideas from chaos theory in numerous fields of science has led to the development of new interest in and new methods of searching for underlying structure and deterministic chaos in time series. Many of these methods have centred around ideas of *scaling* (various concepts of fractal dimension, such as correlation dimension, information dimension, and pointwise dimension) and notions of *embedding* (reconstruction of hidden state space from an observed time series). In this talk we will review these ideas and methods and relate them to notions of chaos, predictability, and determinism in time series.

ANDRÉ DABROWSKI, Department of Mathematics and Statistics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada
A unified approach to fast teller queues and ATM

In the design of modern ATM switches, it is necessary to use simulation to estimate the probability that a queue within the switch exceeds a given large value. Since these are extremely small probabilities, importance sampling methods must be used. Here we obtain a change of measure for a broad class of models with direct applicability to ATM switches.

We consider a model with several independent (not necessarily identical) sources of cells where each source is modeled by a Markov renewal point process with batch arrivals. We do not assume that batch sizes are independent of the state of the Markov process. These arrivals join a queue served by multiple independent servers, each with service times also modeled as a Markov renewal process. The queue is viewed as the additive component of a Markov additive chain constrained so that the additive component remains nonnegative. We apply the theory in McDonald (1996) to obtain the asymptotics of the tail of the distribution of the queue size in steady state and that of the mean time between large deviations of the queue size.

Joint work with B. Beck (Université Catholique de Louvain) and D. McDonald (Ottawa).

ERIC DERBEZ, The Fields Institute
Generating functions and Integrated SuperBrownian Excursion (ISE)

Using the lace expansion (first developed by Brydges and Spencer) as implemented by Hara and Slade, Derbez and Slade have recently proved that lattice trees (studied in statistical mechanics) converge in an appropriate scaling limit to a variant of super Brownian motion known as Integrated SuperBrownian Excursion (ISE). The key to linking these two seemingly disconnected models is a rather simple looking generating function.

SHUI FENG, McMaster University and Fields Institute
The behaviour of some degenerate diffusions near boundary under large deviations

In this talk we will provide answers to the following question: What are the possible paths of some degenerate diffusions under large deviations?

RENÉ FERLAND, Université du Québec à Montréal, Montréal, Québec H3C 3P8, Canada
Propagation of chaos: from Physics to Finance

The chaos propagation property was first proved by Mark Kac for a simple model which bear his name since then. We present chaos propagation results (and fluctuations) for Boltzmann-like equations (scalar and mollified) and for scalar conservation laws. We also discuss possible applications of propagation of chaos in mathematical finance.

A. FÖLDES, The College of Staten Island, CUNY, New York, New York, USA
About the local time of random walk and Brownian motion

The local time of the random walk and the local time of the Brownian motion (just like the processes themselves) share many properties. Strong approximation results will be discussed for the partial sum process of i.i.d. sequence of vectors having dependent components, where the components of the approximating process are independent. Applications of these results for additive functionals of random walk in one and two dimensions and for Brownian motion will also be given. This is a joint work with E. Csáki.

GENEVIÉVE GAUTHIER, École des Hautes Études Commerciales, Service de l'enseignement des methodes quantitatives de gestion, 3000, chemin de la Cote-Sainte-Catherine, Montréal, Quebec H3T 2A7, Canada
Asymptotic distribution of the EMS option

Monte Carlo simulation is a commonly used valuation tool in finance. The method is useful for computing prices of derivative securities when an analytical solution does not exist. Recently, a new simulation technique, known as empirical martingale simulation (EMS), has been proposed by Duan and Simonato (1998) as a way of improving simulation accuracy. EMS has one drawback, however. Because of the dependency among sample paths created by the EMS adjustment, the standard error of the price estimate cannot be obtained by simply using one simulation sample and its asymptotic distribution is unknown. In this paper, we develop a scheme that is capable of overcoming this deficiency. The EMS price estimator is shown to have an asymptotically normal distribution.

EDIT GOMBAY, Department of Mathematical Sciences, University of Alberta, Edmonton, Alberta T6G 2G1, Canada
Correcting some limit theorems about the likelihood ratio

To put the new results into perspective, first, we review some aspects of the history of the likelihood ratio since its definition by Neyman and Pearson in 1928.

Strong approximation of the maximum likelihood ratio statistic by a diffusion process under the null hypothesis is given. This allows one to develop statistics using different weight functions. Sequential tests will be defined and the precision of the approximation is examined.

The asymptotic distribution of the likelihood ratio under noncontiguous alternatives is shown to be normal for the exponential family of distributions. The rate of convergence of the parameters to the hypothetical value is specified where the asymptotic noncentral chi-square distribution no longer holds. It is only a little slower than $O(n-1/2)$. The result provides compact power approximation formulae and is shown to work reasonably well even for moderate sample sizes.

In conclusion, we briefly consider the consequences of these theorems in sequential testing theory.

LAJOS HORVÁTH, Department of Mathematics, University of Utah, Salt Lake City, Utah 84112, USA

Best approximations for bootstrapped processes with applications

We study the asymptotic properties of bootstrapped empirical processes based on "naive" and weighted bootstrap. We show that the order of the best possible approximation for the bootstrapped processes cannot be better than $O_P(n^{-1/2}\log n)$ and we construct sequences of Brownian bridges where this rate is achieved. We discuss applications of the main results to density estimation and change-point detection.

GAIL IVANOFF, Department of Mathematics and Statistics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada

Set-indexed Martingales

We give an appropriate framework and definitions for the theory of set-indexed martingales. Compensators and quadratic variation processes will be defined, and used to develop set-indexed analogues of the classical martingale characterizations of the Poisson process and Brownian motion. Sufficient conditions will be given in terms of compensators for a Poisson convergence theorem and in terms of quadratic variation processes for a martingale central limit theorem. Applications to set-indexed empirical processes will be discussed.

MICHAEL KOURITZIN, University of Alberta, Edmonton, Alberta T6G 2G1, Canada

Parabolic equations with random coefficients

Questions related to the asymptotic behavior (as $\epsilon \to 0$) of systems of random ordinary differential equations

$$\dot{X}^\epsilon(t) = F\Big(X^\epsilon(t), \frac{t}{\epsilon}, \omega\Big), \quad \epsilon > 0,\ X^\epsilon(0) = x_0,$$

where $\{F(x,s), s \geq 0\}$ is a random process for each $x \in \Re^d$, have attracted a multitude of investigations due to applications in such diverse areas as celestial mechanics, oscillation theory, adaptive filtering, recursive identification, and stochastic adaptive control.

A natural question that is important to filtering theory and stochastic control is whether these convergence results continue to hold for the parabolic partial differential equations

$$\partial_t u^\epsilon(x,t) = \sum_{|k|\leq 2p} A_k\Big(x, \frac{t}{\epsilon}, \omega\Big)\partial_x^k u^\epsilon(x,t), \quad \epsilon > 0,\ u^\epsilon(x,0) = \phi(x).$$

For second order parabolic equations with various technical and simplifying assumptions, earlier results indicate that laws of large numbers and fluctuation results continue to hold, provided one resorts to spaces of generalized functions for the fluctuation results. In this talk, we will discuss general convergence and rate of convergence results for u^ϵ. In particular, we will only assume that the coefficients themselves satisfy natural convergence or fluctuation results and we will prove our fluctuation results on a natural Hilbert space. Finally, our setting is general enough to allow for long-range dependence and/or heavy-tail distributions within our work on fluctuations.

REG KULPERGER, Department of Statistical and Actuarial Sciences, University of Western Ontario, London, Ontario N6A 5B7, Canada
Empirical processes and tests of independence

Empirical processes and empirical distributions loose certain information. For example the usual empirical distribution is invariant under a permutation of the data. However some information about the dependence structure can be retained either by the use of random weights or blocks. This talk will consider empirical processes and their use in tests of randomness.

BRENDA MACGIBBON, Département de mathématiques, Université du Québec à Montréal, Montréal, Québec H3C 3P8, Canada
On statistical minimax estimation and principal eigenfunctions of the Laplacian

In many parametric statistical estimation problems, there is definite prior information concerning the values of the parameter vector. There may be bounds on the individual components or on a particular functional of the whole vector. Many computationally feasible estimation methods have been developed to capitalize on such information, but theoretical results have lagged behind. One approach is the worst case analysis: given some error measure, compute the maximum expected error over the restricted parameter space. The study of the resulting best or minimax risk is related here to the study of the classical Dirichlet problem and of principal eigenfunctions of the Laplacian.

NEAL MADRAS, Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada
In search of faster simulations

Markov Chain Monte Carlo has been one of the most active topics in applied probability in recent years. The main idea is that one can generate random samples from a given complicated distribution by first inventing a Markov chain whose equilibrium distribution is the distribution of interest, and then simulating the chain on a computer. Unfortunately, the chain may converge to equilibrium very slowly; for example, this can be the case when the equilibrium distribution is strongly multimodal. One class of remedies involves creating a sequence of overlapping distributions that interpolate between the distribution of interest and some "easy" distribution (whose associated Markov chain equilibriates rapidly). These methods have been successful in practice, but the mathematical theory behind them is not yet complete.

DON L. MCLEISH, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
Estimating parameters of financial time series using highs and lows

Observations on security prices, currency exchange rates interest rates and other financial time series usually include not only open and closing values over periods of time but also highs and lows for the same periods. This information on highs and lows is often disregarded, in part because the joint distributions are very complicated in fact most are unknown). However, these observations are highly informative about the volatily, the correlations among securities, how to hedge one using another, testing for a change in drift, and the valuation of certain options lookback, barrier, *etc.*) that depend on upper and lower barriers. We suggest some semi-parametric methods for using this information efficiently, illustrating the methods using the stock prices of the major Canadian Banks.

MAJID MOJIRSHEIBANI, School of Mathematics and Statistics, Carleton University, Ottawa, Ontario K1S 5B6, Canada
Combined estimation and probabilistic classification

There has recently been a growing interest in combining different classifiers in order to develop more effective classification rules with higher predictive power. The individual classifiers could be, for example, tree classifiers, partitioning rules, Fisher's linear discriminant function, or nearest neighbour classifiers, to name a few. Combining classifiers may also be viewed as a partial answer to the question of: Given a few classification rules, which one should the user choose if his/her main concern is to have a low error rate? Quite often, in a given situation, one classifier performs better than another; the reason can be directly related to the nature of the underlying parent distributions of the classes involved.

In this talk we will propose a new combining procedure which is quite simple to use in practice. We will also show that under certain conditions, the proposed combined classifier is asymptotically, strongly, at least as good as any one of the individual classifiers.

BRUNO REMILLARD, Université du Québec à Trois-Rivières
Empirical processes based on pseudo-observations

Pseudo-observations can be described as estimations or predictions of non observable random variables. In practice, they are used to estimate the distribution function of the non observable random variable. For example, regression residuals are pseudo-observations.

In this talk, I will give some conditions in order that the empirical process based on pseudo-observations converges in law to a continuous stochastic process. Many examples will illustrate the results.

PÁL RÉVÉSZ, Mathematical Institute of Hungary, Academy of Science, Budapest, Hungary
Critical branching Wiener process in the d-dimensional Eucledean space (3 talks)

Consider a particle which executes a critical branching Wiener process. Assume that this particle is living till time T. Then we investigate the empirical distribution function defined by the locations of the particles at time T as well as some functionals of this distribution. A related question is the behaviour of the probability that at least one particle is located in a given ball. We also consider the properties of a critical branching random field. That is we have a Poisson point process in time 0 and the points of this field execute independent branching Wiener processes. A typical question is to investigate the radius of the smallest ball around the origin which consists of at least one particle at time T.

JEFFREY ROSENTHAL, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada
The mathematics of Markov chain Monte Carlo algorithms

Markov chain Monte Carlo (MCMC) algorithms, such as the Gibbs sampler and the Metropolis-Hastings algorithm, are now widely used in statistics, computer science, physics, and chemistry to understand complicated probability distributions. While the implementation of these algorithms is often routine, many fundamental questions—such as convergence rates—are much more difficult. In this talk, we will review some of these issues and the partial progress that has been made towards resolving them.

THOMAS S. SALISBURY, Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada
The complement of the planar Brownian path

Take a planar Brownian path, run until it exits the unit ball. The complement of the path consists of many components, and we can ask about their general shape. The talk will describe joint work with Yuval Peres, in which we show that the components are round, in the sense that their areas are comparable to the square of their diameters. More formally, we show that for every $\beta > 0$, $\sum A_i^\beta < \infty$ if and only if $\sum R_i^{2\beta} < \infty$, where A_i is the area of the i-th component, and R_i is its diameter.

BYRON SCHMULAND, University of Alberta, Edmonton, Alberta T6G 2G1, Canada
Rademacher's theorem on configuration space

Rademacher's theorem says that a Lipschitz function on R^d is differentiable almost everywhere. Recently, Röckner and Schied have proved a version of this result for functions on the configuration space Γ_{R^d} of locally finite subsets of R^d. In this talk, we will explain how this result can be used in the construction and quasi-sure analysis of a diffusion process living on Γ_{R^d}.

QI-MAN SHAO, Department of Mathematics, University of Oregon, Eugene, Oregon 97403, USA
Gaussian correlation conjecture and small ball probabilities

The Gaussian correlation conjecture states that for any two symmetric convex sets in n-dimensional space and for any centered Gaussian measure on that space, the measure of the intersection is greater than or equal to the product of the measures. It is known that the conjecture is true for some special cases, which has been found a very useful tool in the study of the so called small ball probabilities for Gaussian processes. This talk will review recent progress on the conjecture. A new Gaussian correlation inequality as well as its application to the existence of small ball constant for fractional Brownian motion will be discussed.

ZHAN SHI, Laboratoire de Probabilités, Université Paris VI, 4, Place Jussieu 75252 Paris cedex 05, France
The maximum of the uniform empirical process

Some problems—with or without an answer—concerning the maximum of the uniform empirical process will be discussed. This is joint work with Endre Csáki.

GORDON SLADE, Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada
The scaling limit of the incipient infinite cluster in high-dimensional percolation

We describe recent joint work with Takashi Hara proving that, for independent bond percolation in high dimensions, the scaling limits of the incipient infinite cluster's two-point and three-point functions are those of integrated super-Brownian excursion (ISE).

CHRISTOPHER G. SMALL, Pure Mathematics Department, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
The analysis of random shapes

The shape of an object or image, regarded as a subset of $\mathbb{R}^n$, can be defined as the total of all properties which are invariant under similarity transformations of the object in $\mathbb{R}^n$. In practice, the shape of an object is encoded from the configuration of a finite set of points called *landmarks* chosen at important locations on the object. In this talk, we shall survey two of the main approaches to shape analysis due to F. L. Bookstein and D. G. Kendall. Small (1996) provided an extension of the Bookstein representation by representing the shapes of p-simplexes on manifolds. These manifolds are quite distinct from those proposed by D. G. Kendall based upon Procrutes distances. A formula for geodesic distance in simplex shape space permits the implementation of multidimensional scaling methods for the statistical shape analysis of 2- and 3-dimensional objects.

We apply this representation of simplex shapes to a shape analysis of Iron Age brooches from modern day Switzerland. The brooches are then grouped into five classes based upon chronological ordering. It is shown that these five classes form relatively distinct groups when the first two principal coordinates of shape variation are displayed. This supports the assumptions used in archaeological seriation where artifacts are given a rough chronological ordering based upon stylistic features.

BARBARA SZYSZKOWICZ, School of Mathematics and Statistics, Carleton University, Ottawa, Ontario K1S 5B6, Canada
An interplay of weighted approximations and change-point analysis

When studying change-point problems, weighted partial sum-type processes frequently appear as the natural outcome of some nonparametric as well as parametric considerations. We present how approximation methods can lead to obtaining convergence results for such processes and to some "unexpected" results for their sup-functionals. Results and methods of strong and weak approximations have become an integral part of the theory and applications of probability and statistics in the last 40 or so years. Recent contributions in this area are mainly concerned with weighted approximations of stochastic processes based on i.i.d. observations (*cf.*, *e.g.*, M. Csörgő and L. Horváth, Weighted Approximations in Probability and Statistics, Wiley 1993). We construct tools for use in weighted approximations of additive processes in various metrics under i.i.d. sampling and combine these techniques with Le Cam's theory of contiguous measures. An appropriate parametrization of contiguity enables us to quantize a possible change from i.i.d. sampling to small disturbances afterwards in large sets of chronologically ordered data. Constructed new tools allow us to obtain results under the most stringent conditions.

KEITH WORSLEY, Department of Mathematics and Statistics, McGill University, Montreal, Quebec H3A 2K6, Canada

The geometry of correlation fields, with an application to functional connectivity of the brain

Repeated 3D images of brain function, obtained by Positron Emission Tomography (PET) or functional Magnetic Resonance Imaging (fMRI), are now being used to detect pairs of points in 3D that show high functional connectivity. This is defined as the usual sample correlation coefficient measured at the two points in the images, which generates a 6D correlation random field. To detect pairs of regions that are highly correlated, we find tail probabilities of local maxima of the correlation field, and the size of the largest set of connected points in 6D where the correlation field is above a fixed high threshold. The main tool used is the expectation of the Euler characteristic of the excursion set. Results are applied to an experiment to determine which brain regions are functionally connected during an attention task. This is joint work with Jin Cao (Bell Labs, Lucent Technologies).

HAO YU, Department of Statistics and Actuarial Sciences, University of Western Ontario, London, Ontario N6A 5B7, Canada

Weighted Kolmogorov-Smirnov test of stock return distribution

In this talk we demonstrate how Chibisov-O'Reilly theorem be used to do goodness-of-fit test for stock return distributions. The problem related to find the critical values for the test is discussed. The result shows this test is quite effective to deal with heavy tail distributions typically presented in stock return data.

RIČARDAS ZITIKIS, Carleton University, Ottawa, Ontario K1S 5B6, Canada

The Vervaat, Lorenz and some other related processes of probability and mathematical statistics in weighted metrics

We shall discuss various probabilistic properties of the general Vervaat process (that appears in: M. Csörgő and R. Zitikis, *Strassen's LIL for the Lorenz curve.* J. Multivariate Anal. (1) **59**(1996), 1–12) along with its crucial role when analyzing the Lorenz, Bonferroni, Goldie and other related processes of statistical relevance. We shall discuss the importance of investigating the convergence of these processes in weighted supremum metrics, and under minimal assumptions as well.

Topology—4 sub-sessions / Topologie—4 sous-sessions

E. Campbell, Organizer

Muang Min-Oo and McKenzie Wang, Organizers

IGOR BELEGRADEK, Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada
Pinching and Pontrjagin classes

We proof some pinching theorems for the class $\mathcal{M}_{a,b,\pi,n}$ of n-manifolds that have fundamental groups isomorphic to π and that can be given complete Riemannian metrics of sectional curvatures within $[a,b]$ where $a \leq b < 0$. For example, given a word-hyperbolic group π and an integer n there exist $\epsilon = \epsilon(n,\pi) > 0$ such that the tangent bundle of any manifold in the class $\mathcal{M}_{-1-\epsilon,-1,\pi,n}$ has has zero rational Pontrjagin classes.

CHRISTOPH BÖHM, Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada
Inhomogeneous Einstein metrics on spheres

The standard metric on the n-dimensional sphere S^n is Einstein, *i.e.* its Ricci tensor is a multiple of itself. The first non-trivial example of an Einstein metric on S^n was given by Jensen for $n = 4m+3$ in 1973. Five years later Bourguignon and Karcher described a further Einstein metric on S^{15}. Up to now these metrics were the only known examples of Einstein metrics on spheres. All of them are homogeneous and Ziller proved that there exists no further homogeneous Einstein metric on S^n. We describe infinitely many inhomogeneous Einstein metrics with positive scalar curvature on S^5, S^6, S^7, S^8, S^9. The resulting sequence of Einstein metrics converges to an explicit known limit metric which is smooth outside two submanifolds. Hence we get new examples of manifolds where the Palais-Smale condition for the total scalar curvature functional is not fulfiled.

JEFFREY BOLAND, Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada
Magnetic fields on negatively curved manifolds

Closed 2-forms Ω on a Riemannian manifold M induce magnetic field flows in the unit tangent bundle SM which model the motion of a charged particle under the influence of the "magnetic field" Ω. When M is negatively curved and the magnetic field is weak, these flows are hyperbolic and have many interesting properties. We discuss in particular their entropy, the regularity of their hyperbolic splitting, and the question of when they are isomorphic to (Finsler or Riemannian) geodesic flows. Particular attention will be given to the case when M is a locally symmetric space.

VIRGINIE CHARETTE, University of Maryland, College Park, Maryland 20742, USA
Properly discontinuous actions of free groups on Minkowski space

Margulis showed that free, non-abelian groups can act freely and properly discontinuously on three-dimensional Minkowski space. These examples of "affine Schottky groups" came as a surprise.

The talk will focus on quotients of Minkowski space by such groups of isometries that can be factored as products of inversions. Certain preferred fundamental polyhedra for their action will be introduced and facts about recurring geodesics will be presented.

JINGYI CHEN, Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
Triholomorphic curves and complex ASD connections

We study the adiabatic limit of the complex anti-self-dual connections over a product of two Calabi-Yau surfaces and the triholomorphic curves between kyperkähler manifolds.

AILANA FRASER, Courant Institute, New York, New York 10012, USA
On the free boundary variational problem for minimal disks

We will discuss the problem of extremizing the energy (equivalently area) for maps from the unit disk D into a Riemannian manifold N having boundary lying on a specified embedded submanifold M. The critical points of this geometric variational problem are minimal surfaces which meet the submanifold orthogonally along the boundary. We derive a partial Morse theory for this problem in arbitrary dimensions. In addition, we use the geometry to obtain certain lower bounds on the Morse index for such disks.

MICHAEL GAGE, University of Rochester, Rochester, New York 14627, USA
Remarks on B. Süssmann's proof of the Banchoff-Pohl inequality

This is an expository talk describing Bernd Suessmann's use of the curve shortening flow to prove the Banchoff-Pohl isoperimetric inequality for non-simple closed curves on simply connected surfaces with Gauss curvature bounded above by a non-positive constant K_0. The inequality is

$$L^2 - 4\pi \int_M w(x)^2 \, dA(x) + K_0 \Big(\int_M |w(x)| \, dA(x) \Big)^2 \geq 0$$

were L is the length of the curve γ and $w(x)$ is the winding number of γ about the point x. The idea of the proof is to show that the left hand side cannot be increased under the curve shortening flow. This is sufficient because the curve shortening flow deforms an arbitrary closed curve to a "circular" point (the curve may temporarily develop cusps during process) and the left hand side is non-negative for curves near these points.

The inequalities Süssmann derives in order to prove that this quantity decreases under the curve shortening flow are interesting and probably more powerful than the final result.

MIROSLAV LOVRIĆ, Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada
Multivariate normal distributions parametrized as a Riemannian symmetric space

The construction of a distance function between probability distributions is of importance in mathematical statistics and its applications. Distance function based on the Fisher information metric has been studied by a number of statisticians, especially in the case of the multivariate normal distribution (Gaussian) on $\mathbf{R}^n$. It turns out that, except in the case $n = 1$, where the Fisher metric describes the hyperbolic plane, it is difficult to obtain an exact formula for the distance function (although this can be achieved for special families with fixed mean or fixed covariance). We propose to study a slightly different metric on the space of multivariate normal distributions on $\mathbf{R}^n$. Our metric is based on the fundamental idea of parametrizing this space as the Riemannian symmetric space $\mathrm{SL}(n+1)/\mathrm{SO}(n+1)$. Symmetric spaces are well understood in Riemannian geometry, allowing us to compute distance functions and other relevant geometric data.

MOHAN RAMACHANDRAN, SUNY at Buffalo, Buffalo, New York 14214-3093, USA
To be announced

PATRICK RYAN, McMaster University, Hamilton, Ontario L8S 4K1, Canada
Real hypersurfaces in complex space forms

We consider hypersurfaces immersed in a complex projective space $\mathbf{C}P^n$ or complex hyperbolic space $\mathbf{C}H^n$—the space forms of nonzero constant holomorphic sectional curvature. We are interested in characterizing certain standard examples (*e.g.* the homogeneous ones) in terms of intrinsic and extrinsic geometrical properties. Recent results and open problems in this area will be discussed.

PATRICE SAWYER, Department of Mathematics and Computer Science, Laurentian University, Sudbury, Ontario P3E 2C6, Canada
Ghostly symmetric spaces

Various objects associated with symmetric spaces have specific ranges. We are thinking in particular of root multiplicities in the root system (nonnegative integers) and of the Cartan subalgebra ($\mathbf{R}^l$). We will see that by extending these ranges and thus going beyond the "physical" symmetric spaces (hence the title), we can solve problems concerning ordinary symmetric spaces.

ALINA STANCU, Courant Institute of Mathematical Sciences, New York, New York 10012, USA
Asymptotic behavior of a crystalline evolution

Motion by crystalline curvature is viewed as a typical example of geometric evolution by a nonsmooth boundary energy. Assume that a planar curve is endowed with an energy density defined on a finite set of normal directions. It is natural then to consider the restricted class of piecewise linear curves with just this ordered set of normals. These curves do not have a motion by curvature in the conventional geometric sense, but, following M. Gurtin and J. Taylor, one can still define the so-called crystalline curvature flow which is analogous to the motion by weighted curvature for smooth planar curves.

We consider Gurtin's defintion, with no driving term, for closed convex curves, so that the inward normal velocity of each segment of an admissible polygonal curve as above is inversely proportional to the length of the segment, where the proportionality factor is only required to be positive. Our results show that, if the curve has more than four sides, it will shrink to a point while approaching the shape of a homothetic solution to the flow. This implies the existence of at least one self similar solution for any flow associated to an energy density defined on more than four unitary directions. The number of homothetic solutions will be discussed based on the properties of the energy density.

JOHN TOTH, McGill University, Montreal, Quebec H3A 2K6, Canada
To be announced

2) Homotopy Theory / Théorie de l'homotopie

Lisa Langsetmo and Jim Shank, Organizers

DAN CHRISTENSEN, Johns Hopkins University, Baltimore, Maryland 21218, USA
Phantom maps: all or nothing

The general theme is to determine conditions on spectra X and Y under which either every map from X to Y is phantom, or no non-zero maps are. I will also address the question of whether such all or nothing behaviour is preserved when X is smashed with a finite spectrum W. It turns out that there are close connections with the divisibility and rationality of the group $[X, Y]$, and with Brown-Comenetz duality.

If there is time, I will discuss n-phantom maps, a chromatic analog of phantom maps, and use this concept to give another explanation of the results on phantom maps.

This is joint work with Mark Hovey and Sharon Hollander.

F. R. COHEN, Department of Mathematics, University of Rochester, Rochester, New York 14627, USA
On stunted projective spaces

F. Sergeraert and V. Smirnov have recently studied the homology of the loop spaces for $RP^\infty/RP^n = X_n$. It is the purpose of this note to record the homotopy type of these spaces after looping (sometimes more than once). The following is joint work with R. Levi.

Theorem. *(1) $\Omega(X_1)$ is homotopy equivalent to $S^1 \times \Omega S^3$.*

(2) $\Omega(X_2)$ is the homotopy theoretic fibre of a map $S^3 \to Z$ where Z denotes the 6-skeleton of the Lie group G_2. Thus $\Omega_0^4(X_2)$ is homotopy equivalent to $\Omega_0^3 S^3 \times \Omega_0^4(Z)$.

(3) $\Omega(X_3)$ is homotopy equivalent to $S^3 \times \Omega\big(S^7 \vee P^6(2)\big)$.

(4) $\Omega(X_n)$ is the homotopy theoretic fibre of a map from a finite complex to the Lie group $Spin(n)$.

Further information concerning the map $X_n \to \Sigma RP^n$ is given. For example, the theorem above for X_2 gives a different proof of a theorem of Jie Wu concerning a splitting of the homotopy groups for ΣRP^2.

GUSTAVO GRANJA, Department of Mathematics, MIT, Cambridge, Massachusetts 02139, USA
On self maps of HP^n

In 1975, Feder and Gitler computed the action of the Adams operations on the K-theory of HP^n and used this to get restrictions on the possible degrees of self maps. They conjectured that all the integers satisfying these restrictions are degrees of actual self maps. The conjecture is known to be true for $n = 2, 3$ and ∞. We show that it also holds for $n = 4, 5$ by using computations by

Curtis and Mahowald of the unstable Adams spectral sequence for S^3 and explain why this gives qualitatively stronger evidence for the conjecture.

STEVE HALPERIN, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada

The homotopy Lie algebra of a finite complex

Sullivan models provide an effective means of using finiteness hypotheses on homology to deduce non-trivial properties of the homotopy Lie algebra. This talk presents a survey, including some new results joint with Y. Felix and J.-C. Thomas.

PHILIP R. HEATH, Department of Mathematics, Memorial University of Newfoundland, St. John's, Newfoundland A1C 5S7, Canada

Fibre techniques in Nielsen periodic point theory

Fibre techniques have been used in ordinary Nielsen fixed point theory since the late sixties. In this talk I indicate how recent developments allow fibre techniques to be use in Nielsen periodic point theory. Applications to nilmanifolds and solvmanifolds (including the Klein Bottle) are forthcoming.

BARRY JESSUP, Department of Mathematics and Statistics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada

Estimating the rational LS-category of elliptic spaces

An *elliptic* space is one whose rational homotopy and rational cohomology are both finite dimensional. Using the equality of rational LS category and Toomer's invariant for elliptic spaces, established by Félix, Halperin and Lemaire, we provide new lower bounds for the rational category of elliptic spaces. We do this by proving two special cases of an even dimensional analogue of a theorem which improved the estimate of the Mapping theorem in some settings. Several interesting examples are presented to illustrate the results, which are joint work with Sonia Ghorbal.

BRENDA JOHNSON, Department of Mathematics, Union College, Schenectady, New York 12308, USA

Constructing and characterizing degree n functors

Let F be a functor from a basepointed category with finite coproducts to a category of chain complexes over an abelian category. Such a functor is homologically degree n if its $n+1$-st cross effect (in the sense of Eilenberg and Mac Lane) has trivial homology. We describe a method for constructing, by means of cotriples associated to the cross effects of F, a universal tower under F,

$$\cdots \to P_{n+1}F \to P_nF \to P_{n-1}F \to \cdots \to P_1F \to P_0F = F(*),$$

in which each functor P_nF is homologically degree n. This construction arose from the study of Goodwillie's Taylor tower in the case of functors of modules over a ring. Using this model, we will characterize homologically degree n functors in terms of modules over a certain DGA, and discuss some related constructions and examples due to Eilenberg-Mac Lane, and Dold-Puppe. This is joint work with Randy McCarthy.

KEITH JOHNSON, Department of Mathematics, Statistics, and Computing Science, Dalhousie University, Halifax, Nova Scotia B3H 3J5, Canada

Elliptic homology cooperations

In this talk I will survey three approaches to describing the hopf algebroid $\mathrm{Ell}_*\mathrm{Ell}$ of stable cooperations for elliptic homology and some of the information about the E_2 term of the Adams spectral sequence based on Ell which results.

SADOK KALLEL, Department of Mathematics, University of British Columbia, Vancouver, British Columbia V6T 1Z2, Canada
The homology structure of free loop spaces

We present recent results on the structure of fibrations with section (joint with D. Sjerve), and give some applications to the theory of free loop spaces. We show how higher differentials on spherical classes in the Serre spectral sequence for the evaluation fibration associated to $\mathrm{Map}(S^k, M)$ are given in terms of certain homology operations in the fiber. As an illustration of our methods, we discuss the example of free loops on configuration spaces (this is a space of fundamental interest in non-linear analysis).

KATHRYN LESH, University of Toledo, Toledo, Ohio 43606, USA
Progress toward a partial splitting of E_2 in the UASS for SO

In the 1970s, Bousfield conjectured a splitting for the E_2 term of the unstable Adams spectral sequence (UASS) for SO. We discuss recent progress towards a proof of the first stages of Bousfield's conjecture.

L. GAUNCE LEWIS, JR., Mathematics Department, Syracuse University, Syracuse, New York 13244-1150, USA, current address: Mathematics Department, MIT, Cambridge, Massachusetts 02139, USA (on leave for 1998–99)
Recent results on Mackey functors for a compact Lie group

Mackey functors were first introduced as a tool for proving induction theorems in representation theory. They have, however, become an important tool in equivariant homotopy theory because any reasonable equivariant cohomology theory is implicitly Mackey functor valued. For applications in equivariant homotopy theory, it would be very nice to have a well-behaved extension of the notion of a Mackey functor from its original context of finite groups to the context of compact Lie groups. Unfortunately, various technical difficulties have so far severely limited the utility of the available extensions. However, the new approach to Mackey functors for a finite group taken recently by Florian Luca turns out to extend nicely to compact Lie groups. The effectiveness of this extension of his methods to compact Lie groups will be illustrated in this talk by presenting its implications for the structure of the spectrum of a commutative Mackey functor ring.

JOHN MARTINO, Western Michigan University, Kalamazoo, Michigan 49008, USA
A Minami-Webb formula for compact Lie groups

The Minami-Webb formula expresses the classifying space of a finite group G as the linear combination,

$$BG = \sum_H \alpha_H H,$$

of mod -p cyclic groups (*i.e.*, extensions of p-groups by cyclic p'-groups). The formula and its variants have proven very useful in calculating group cohomology. We generalize the Minami-Webb formula to compact Lie groups.

JOSEPH NEISENDORFER, University of Rochester
James-Hopf invariants, Anick's spaces, and decompositions of the double loops on a Moore space

Using spaces introduced by Anick, we construct a decompostion into indecomposable factors of the double loop spaces of odd primary Moore spaces when the powers of the primes are greater than the first power. If n is greater than 1, this implies that the odd primary part of all the homotopy groups of the $2n+1$ dimensional sphere lifts to a mod p^r Moore space.

STEWART PRIDDY, Department of Mathematics, Northwestern University, Evanston, Illinois 60208, USA
Decomposing products of classifying spaces

Let $G = H \times K$ be the product of two finite groups. The problem of determining the stable type of the classifying space $BG = BH \times BK$, localized at a prime p, is often quite difficult due to the complicated nature of the subgroups of G which generally do not relate well to those of H and K. Of course, the usual stable decomposition of a product of spaces $X \times Y \cong X \vee Y \vee (X \wedge Y)$ gives a decomposition of BG but these summands may be decomposable, even in elementary cases such as $B(\mathbf{Z}/2 \times \mathbf{Z}/2)$. In some situations however, the stable type of BG is closely related to that of BH and BK. We say BG is *smash decomposable* if the indecomposable summands of BG_+ are given by simply smashing together those of BH_+ and BK_+, that is, a complete splitting for BG_+ localized at p has the form

$$BG_+ = BH_+ \wedge BK_+ \cong \bigvee_{i,j} X_i \wedge Y_i$$

where $BH_+ \cong \bigvee_i X_i$, $BK_+ \cong \bigvee_j Y_j$ are complete splittings into indecomposable summands. Here BG_+ is BG with a disjoint basepoint. This is convenient for converting from Cartesian products to smash products since $(X \times Y)_+ = X_+ \wedge Y_+$. Further $BG_+ = BG \vee S^0$. Thus BG is smash decomposable when the $X_i \wedge Y_j$ are indecomposable. In this talk we study groups for which BG is smash decomposable.

(Joint work with John Martino and Jason Douma)

CHARLES REZK, Northwestern University, Evanston, Illinois 60208, USA
A model for the homotopy theory of homotopy theory

We describe a category, the objects of which may be viewed as models for homotopy theories. We show that for such models, "functors between two homotopy theories form a homotopy theory", or more precisely that the category of such models has a well-behaved internal hom-object. This set-up has applications to problems of realizing diagrams up to homotopy by "honest" diagrams.

LAURA SCULL, Department of Mathematics, University of Chicago, Chicago, Illinois 60637, USA
Rational S^1-equivariant homotopy theory

I will discuss an algebraicization of rational S^1-equivariant homotopy theory. There is an algebraic category of "T-systems" which is equivalent to the homotopy category of rational S^1-simply connected S^1-spaces. There is also a theory of "minimal models" for T-systems, analogous to

Sullivan's minimal algebras. Each S^1-space has an associated minimal T-system which encodes all of its rational homotopy information, including its rational equivariant cohomology and Postnikov decomposition.

PAUL SELICK, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada
Natural decompositions of loop suspensions and tensor algebras

(Joint work with Jie Wu).

Consider the full subcategory of pointed topological spaces whose objects are simply connected suspensions of finite type. For X in the above category we examine natural decompositions of $\Omega\Sigma X$ localized at a prime p as a product (up to homotopy) of other spaces. Since as a Hopf algebra, $H_*(\Omega\Sigma X\ ;\ \mathbb{Z}/p\mathbb{Z})$ is isomorphic to the tensor algebra $T(V)$, where $V = \tilde{H}_*(X\ ;\ \mathbb{Z}/p\mathbb{Z})$, any such decomposition yields a natural coalgebra decomposition of $T(V)$ (which need not be a Hopf algebra decomposition since we have not required our decomposition to respect the H-space structure on $\Omega\Sigma X$). We have shown that the converse is true: every natural decomposition of $T(V)$ can be geometrically realized as a natural decomposition of the space $\Omega\Sigma X$. Having thus translated the problem to algebra, we next consider the algebraic problem of finding natural coalgebra decompositions of tensor algebras. We show that there is a natural coalgebra decomposition of $T(V)$ (natural with respect to the vector space V) $T(V) = A^{\min}(V) \otimes B_2(V) \otimes B_3(V) \cdots$ where $A^{\min}(V)$ contains V itself and is minimal in the sense that it is (up to isomorphism) a retract of any coalgebra containing V which is a natural retract of $T(V)$. The coalgebra $B_n(V)$ is the smallest natural coalgebra retract containing a certain submodule $L_n^{\max}(V)$ described below. This decomposition generalizes that given by the Poincaré-Birkhoff-Witt Theorem, except it is natural with respect to maps of vector spaces, whereas PBK is natural only with respect to maps of ordered vector spaces.

Some properties of $A^{\min}(V)$ and of the product $B(V) = B_2(V) \otimes B_3(V) \cdots$ of all the other factors are as follows. $B(V)$ is a sub-Hopf-algebra of $T(V)$ which is a retract as a coalgebra. We show that, as conjectured by Cohen, the only primitives in $A(V)$ occur in weights of the form p^t. Also, $A(V)$ has a filtration where each of the filtration quotients is a polynomial algebra. A description of the generators for these polynomial algebras is given for the first $p^2 - 1$ filtration quotients, computation of the others remaining beyond our present capabilities.

One important aspect of this work is its relationship to the $\bmod p$ representation theory of the symmetric group Σ_n. It provides some information about the important Σ_n-module $\mathrm{Lie}(n)$ described below which has arisen in many contexts and appears in current work of Cohen, Dwyer, Arone, and others. To define $\mathrm{Lie}(n)$ consider the vector space V with basis $\{v_1, \ldots, v_n\}$. There is an action of Σ_n on (V) (and thus on $T(V)$) given by $v_j \mapsto v_{\sigma(j)}$ for $\sigma \in \Sigma_n$. Let $\gamma(V)$ be the subspace of $V^{\otimes n}$ spanned by $\{v_{\sigma(1)} \cdots v_{\sigma(n)}\}$. Let $L_n(V)$ be the primitives of "weight" n in $T(V)$ which are indecomposable (*i.e.* not p-th powers). Explicitly $L_n(V)$ consists of commutators of length n in the elements of V. Let $L_n^{\max}(V) = L_n(V) \cap B$. Let $\mathrm{Lie}(n) = L_n(V) \cap \gamma$ and let $\mathrm{Lie}^{\max}(n) = L_n^{\max}(V) \cap \gamma$. We show that $\mathrm{Lie}^{\max}(n)$ is a projective Σ_n-submodule of $\mathrm{Lie}(n)$ and that any projective Σ_n-submodule of $\mathrm{Lie}(n)$ is a retract (up to isomorphism) of $\mathrm{Lie}^{\max}(n)$. If n is invertible modulo p then it is well known that $\mathrm{Lie}(n)$ is itself projective and easy to see that $L_n^{\max}(V) = L_n(V)$. In particularly, in characteristic 0, $L_n^{\max}(V) = L_n(V)$ for all n.

STEPHEN D. THERIAULT, Department of Mathematics, MIT, Cambridge, Massachusetts 02139, USA
Homotopy exponents for certain $\bmod\text{-}2^r$ *Moore spaces*

For a prime p, the mod-p^r Moore space $P^m(p^r)$ is the cofiber of the degree p^r map on S^m. A special case of Barratt's conjecture is that if $p^r \neq 2$ then the p-primary torsion in $\pi_*(P^m(p^r))$ is annihilated by p^{r+1}. It is well known that this is true when p is an odd prime. We investigate to what extent it is true when $p = 2$ and $r \geq 2$.

Juris Steprans and Steve Watson, Organizers

MURRAY BELL, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

Cardinal functions of centered spaces

For a collection of sets S, give $\mathrm{cen}(S) = \{T \subset S : T \text{ is centered}\}$ the compact Hausdorff topology that it inherits as a subspace of $2S$. The spaces $\mathrm{cen}(S)$ are exactly the Adequate Compact spaces of Talagrand. They have served topologists well as a rich source of examples. This talk is concerned with spaces which are by definition those Hausdorff spaces that are continuous images of some $\mathrm{cen}(S)$. This is a good generalization of the dyadic spaces. Our focus will be on the relationships between the 9 popular cardinal functions of w, π, t, χ, d, c, $\pi\chi$, $d\chi$ and $d\pi\chi$ for this class of spaces.

MAXIM R. BURKE, University of Prince Edward Island

Continuous functions which take a somewhere dense set of values on every open set

We study the class of Tychonoff spaces that can be mapped continuously into the real line in such a way that the preimage of every nowhere dense set is nowhere dense. We show that every metric space without isolated points is in this class. We also give examples of spaces which have nowhere constant continuous maps into the real line and are not in this class.

KRZYSZTOF CIESIELSKI, West Virginia University

Each Polish space cocompactly quasimetrizable

On April of 1998 Ralph Kopperman and Bob Flagg asked me whether every for Polish space there exists a countable collection C of closed subsets of X such that:

(1) each subset of C with the finite intersection property has nonempty intersection,

(2) for every open set T and x from T there exists a C in C such that C is a subset of T and x belongs to the interior of C, and

(3) for every C from C and x from the complement of C there exists a finite subcollection G of C such that C is contained in the interior of the union U of G and x is still in the complement of U.

I was able to answer this question positively. In fact, the constructed family satisfies condition (3) with the singleton families G. This fact implies, in particular, that the following properties are equivalent for every topological space X.

(A) X is a Polish space.

(B) X is cocompactly quasimetrizable, that is, X is a Lindelf space arising from a quasimetric whose dual yields a compact (not necessarily T_2) topology.

(C) X has a bounded complete approximation (computational) model, that is (loosely), the points can be encoded and approximated by sets containing them in a computer program.

W. W. COMFORT, Wesleyan University

Continuous cross sections on abelian groups equipped with the Bohr topology

All groups here are Abelian. A closed subgroup H of a topological group G is a *ccs-subgroup* if there is a continuous cross section from G/H to G—that is, a continuous function Γ such that $\pi\circ\Gamma=\mathrm{id}|_{G/H}$ (with $\pi\colon G\to G/H$ the natural homomorphism).

The symbol $G^{\#}$ denotes G with its Bohr topology, *i.e.*, the topology induced by $\mathrm{Hom}(G,\mathbb{T})$.

A topological group H is *an absolute ccs-group* (#) [resp., *an absolute retract* (#)] if H is a ccs-subgroup [resp., is a retract] in every group of the form $G^{\#}$ containing H as a (necessarily closed) subgroup. One then writes $H\in\mathrm{ACCS}(\#)$ [resp., $H\in\mathrm{AR}(\#)$].

Theorem 1. *Every ccs-subgroup H of a group of the form $G^{\#}$ is a retract of $G^{\#}$ (and $G^{\#}$ is homeomorphic to $(G/H)^{\#}\times H^{\#}$); hence $ACCS(\#)\subseteq AR(\#)$.*

Theorem 2. *$H^{\#}\in ACCS(\#)$ [resp., $H^{\#}\in AR(\#)$] iff $H^{\#}$ is a ccs-subgroup of its divisible hull $\left(\mathrm{div}\,(H)\right)^{\#}$ [resp., $H^{\#}$ is a retract of $\left(\mathrm{div}\,(H)\right)^{\#}$].*

Theorem 3. *(a) Every cyclic group is in $ACCS(\#)$.*

(b) The classes $ACCS(\#)$ and $AR(\#)$ are closed under finite products.

Theorem 4. *Not every Abelian group is in $ACCS(\#)$.*

Question [van Douwen, 1990]. Is every Abelian group in AR(#)?

* Presented in Kingston by this co-author.

ILIJAS FARAH, York University, Toronto, Ontario M3J 1P3, Canada

Exp(N^*) need not be an N^*-image

We prove, using an idea due to M. Bell, that the Čech-Stone remarinder of the integers does not map onto its exponential space. This result uses the Open Coloring Axiom, OCA, and it is another instance of the effect of OCA to structures closely related to real numbers which is opposite to the effect of Continuum Hypothesis (see [1]).

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DOUGLAS L. GRANT, University of College of Cape Breton

Alternative universes: the role of set theory in topological algebra

This survey will explore some of the theorems of topological algebra in which significantly more powerful results are available than in the analagous case for Hausdorff spaces. In addition, some cases will be considered where the construction of counterexamples has required more powerful set-theoretic assumptions than is the case in general topology. In most of those instances, solutions without set-theoretic axioms beyond ZFC remain open problems.

GARY GRUENHAGE, Auburn University, Auburn, Alabama 36849-5310, USA

More on a-Toronto spaces

A Toronto space is an uncountable non-discrete Hausdorff space which is homeomorphic to every one of it's uncountable subspaces. It is known that a Toronto space, if one exixts, is scattered of Cantor-Bendixon rankw_1 with each level countable. J. Steprans defines an a-Toronto space, where a is an ordinal, to be a scattered space of rank a which is homeomorphic to each subspace of the same rank. We will discuss results related to our recent proof that, consistently, there are countable a-Toronto spaces for any $a < w_1$; for example, we show that the proof can be modified to obtain, for any cardinal k, a k-Toronto space in which each level has cardinality k.

MELVIN HENRIKSEN, Harvey Mudd College, Claremont, California 91711, USA
Embedding a ring of continuous functions in a regular ring: preliminary report

$C(X)$ denotes the ring of continuous real-valued functions on a Tychonoff space X. If $f \in C(X)$, let $f'(x) = \frac{1}{f}(x)$ when $f(x) = 0$, and let $f(x) = 0$ otherwise. Let $G(X)$ denote the subalgebra of $\mathbb{R}_X$ generated by $C(X)$ and $\{f' : f \in C(X)\}$. Then $G(X)$ is the smallest von Neumann regular sublagebra of $\mathbb{R}_X$ containing $C(X)$ and is closed under inversion, *i.e.*, any element of $G(X)$ that never vanishes has an inverse in $G(X)$. Let $Ba_1(X)$ denote that family of pointwise limits and of sequences of elements of $C(X)$, $G^u(X)$ the family of uniform limits of sequences of elements of $G(X)$, and $X\delta$ the topological space obtained by taking the zerosets of $C(X)$ as a base for a topology on X. Then $C(X) \subset G(X) \subset G^u(X) \subset Ba_1(X) \subset C(X\delta)$. Each of these inclusions can be proper, each of these families are vector lattices under the usual pointwise operations, and each of them are algebras with the possible exception of $G^u(X)$. Each $f \in G(X)$ is continuous on an open dense subspace of X, and if X is a Baire space then each $f \in G^u(X)$ is continuous on a dense $G\delta$ of X. The bounded elements of $G^u(X)$ are closed under multiplication, each $f \geq 1$ in $G^u(X)$ is invertible, and $G^u(X)$ is an algebra iff $f^2 \vee 1 \in G^u(X)$ whenever $f \in G^u(X)$. It is not known whether $G^u(Q)$ (where Q is the space of rational numbers) is an algebra, but this latter is not closed under inversion. The relationship between some of these vector lattices and the complete ring of quotients and the epimorphic hull of $C(X)$ is studied by making use of results on the latter due to R. Raphael and R. G. Woods that are not as yet published. Indeed, the present research is joint work with these two authors.

JAMES HIRSCHORN, University of Toronto, Toronto, Ontario M5S 3G3, Canada
Towers of measurable functions

We give mathematical reformulations of the cardinals $\mathfrak{p}$ and $\mathfrak{t}$ in terms of families of measurable functions, proving that both of these cardinals are invariant under the addition of one random real. We also describe the cardinal $\mathfrak{f}$ after adding one random real.

VALERY MISKIN, Kemerovo State University
Set ideals everywhere

Although set ideals are objects of algebraic nature they penetrate like comets many branches of mathematics from Set Theory and Combinatorics to Topology, Analysis, Measure, Ergodic and Number Theory. They appears naturally in these settings and adorn the tree of Mathematics just as the lights of Christmas lighting strings connecting old and new mathematical concepts and problems. As to the ideals arising in Set Theory and Infinite Combinatorics I concentrate on the classical Commutative Algebra concept of quotient of two set ideals (which is not yet a well known instrument in Set Theory and Topology) and related cardinal invariants of set ideals. This concept proved to be crucial in Boolean ring contexts while solving the problem of isomorphism of symmetry groups of set ideals and the problem of description of set ideals with complete or maximal symmetry groups (problem of H. Macpherson and P. Neumann) and in general while studding the automorphisms and normal subgroup lattice of their symmetry groups. I discuss the solutions to these problems and related open problems. As to the properties of set ideals of topological origin I discuss the concepts related to Stone-ech compactification of w and state a generalization of a

Banach-Kuratowski theorem in terms of imprimitivity domains and provide some nice properties of the automorphism group of an absolutely homogeneous countably separated Borel space. I mention some resently posed open problems particularly related to meager and null sets on the real line.

JUSTIN TATCH MOORE, University of Toronto, Toronto, Ontario M5S 3G3, Canada
A linearly fibered Souslinean space Under Martin's axiom

We will sketch the construction, under $MA + \neg CH$, of a c.c.c. nonseparable compact space which maps continuously into the Real line with linear fibers. Such a space can not, for instance, map onto $[0,1]^{\omega_1}$.

ALEXANDER SHIBAKOV, Tenessee Technological University
Controlling sequential order in topological vector spaces

We discuss methods of constructing topological vector spaces with prescribed convergence properties. A possible strategy for making the examples locally convex is also considered. At the end, some open questions will be mentioned.

SLAWOMIR SOLECKI, Indiana University
Polish group actions and measures

The talk will be about a connection between continuous actions of Polish groups and the structure of the equivalence relation of mutual absolute continuity among Borel probability measures defined on a Polish space. We will show that this equivalence relation is induced by a special kind of a continuous action of a Polish group. Background and consequences of this result will be presented. For example, a theorem of Kechris and Sofronidis (that it is not possible to classify, by "simple objects" and in Baire measurable fashion, Borel probability measures up to mutual absolute continuity) will be deduced.

PAUL J. SZEPTYCKI, Ohio University
Normality and property (a)

A space X is said to have property (a) if for each open cover U and each dense $D \subseteq X$, there is a closed discrete $E \subseteq D$ such that $st(E,U) = \bigcup\{u \in U : u \cap E \neg \emptyset\} = X$. This property was recently introduced by M. Matveev and is of particular interest when X is countably compact. Although not a priori obvious, property (a) is closely related to normality. I will discuss recent theorems, examples and open questions.

ANDRZEJ SZYMANSKI, Slippery Rock University of Pennsylvania
On a class of special Namioka spaces

If $f\colon X \times Y \to M$ is a separately continuous function on the product of a compact space Y and a strongly countably complete space X into a metric space M, then there exists a dense $G\delta \subset A$ of X such that f is jointly continuous at each point of $A \times Y$.

We introduce a game-theoretic description of a class of topological spaces that is substantially wider than, for example, the class of strongly countably complete spaces or the class of metric Baire spaces, yet the conclusion of the Namioka theorem holds for spaces from this class. The class itself is also closed under perfect preimages. We apply our results to the theory of semitopological groups.

FRANK TALL, University of Toronto, Toronto, Ontario M5S 3G3, Canada
The topology of elementary submodels

Results and problems concerning the topology of elementary submodels will be discussed.

MURAT TUNCALI, Nipissing University
On generalizations of the Hahn-Mazurkiewicz theorem

The Hahn-Mazurkiewicz theorem characterizes the Hausdorff continuous images of $[0,1]$ as the class of locally connected metric continua (Peano continua). A theorem of Alexandroff gives a characterization of the Hausdorff continuous images of the Cantor ternary set as the class of compact metric spaces. Following these theorems, it was natural to ask whether one could obtain generalizations of these results in the category of Hausdorff spaces.

Nikiel (1988) obtained a characterization of locally connected continuous images of compact ordered spaces. Bula and Turzanski (1986) gave a characterization of conitnuous images of compact ordered spaces. Since these characterizations were obtained, there has been a considerable amount of development in the study of continuous images of ordered continua (arcs) and compact ordered spaces. In this talk, we will give a survey of the results concerning the classes of spaces that are continuous images of ordered continua and compact ordered spaces, and related classes.

E. D. TYMCHATYN, University of Saskatchewan, Saskatoon, Saskatchewan S7N 5E6, Canada
Measures and topological dynamics on Menger manifolds

Joint research with H. Kato, K. Kawamura and M. Tuncali.

We study non-atomic, locally positive, Lebesgue-Stieltjes measures on compact, connected, Menger manifolds. We show that each such manifold X admits an essentially unique, normalized, non-atomic, locally positive, Lebesgue-Stieltjes measure. The set of ergodic homeomorphisms on X forms a dense G_d in the space of all measure preserving autohomeomorphisms of X in the compact open topology. In particular, there exists a topologically transitive homeomorphism on X. We also prove the existence of chaotic homeomorphisms on X.

GRANT WOODS, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada
Recent developments in rings of continuous functions

In 1960 Gillman and Jerison's text "Rings of Continuous Functions", one of the most influential books on general topology ever written, was published. Thirty-eight years later researchers are still investigating the structure of the ring $C(X)$ of continuous real-valued functions defined on a Tychonoff topological space.

In this talk I review recent work by myself and others on three different topics concerning $C(X)$:

(1) The weak topology on the set $X \times Y$ induced by the family of separately continuous functions on the product $X \times Y$ of Tychonoff spaces X and Y.

(2) Regular rings of quotients of $C(X)$, particularly its epimorphic hull.

(3) The relationship among the cardinal numbers $|C(X)|$, $|C(Y)|$, and $|C(X \times Y)|$.

EDUARDO SANTILLAN, York University, Toronto, Ontario M3J 1P3, Canada
and Cinvestav, Mexico DF
Topological properties of removable singularities for analytic functions

A closed set X of a complex manifold M is said to be a *removable singularity* if every analytic function defined on M-X has got an analytic extension to M. In his doctoral tesis, Shiffman proved a partial characterisation of the removable singularities by using the Hausdorff measure. Moreover, in 1994, professors Chirca, Stout and Lupacciolu showed more results which are completely independet of those of Shiffman; they used the cover dimension. The actual challenge is to improve that results to characterise the removable singularities by using just topological properties.

4) Symplectic/Low Dimensional Topology / Topologie en basses dimensions/Topologie symplectique

Steve Boyer, Jacques Hurtubise and Francis Lalonde, Organizers

STEVEN BOYER, Département de mathématiques, Université du Québec à Montréal, Montréal, Québec H3C 3P8, Canada

Norm duality and hyperbolic 3-manifolds

The $\mathrm{SL}_2(C)$-character variety of a 1-cusped hyperbolic 3-manifold has given rise to two convex, balanced plane polygons—the unit ball of the Culler-Shalen norm and the Newton polygon of the (two-variable) A-polynomial of the manifold. In joint work with Xingru Zhang, we show that these polygons are dual in the sense that the line segments joining antipodal vertices of one are parallel to the the sides of the other. Indeed, we show that the Newton polygon can be thought of as a ball of the norm dual to the Culler-Shalen norm. This duality was one of the key ideas in our recent proof of the finite surgery conjecture.

JACQUES HURTUBISE, McGill University, Montreal, Quebec H3A 2K6, Canada

To be announced

FRANCOIS LALONDE, University of Quebec at Montreal, Montreal, Quebec H3C 3P8, Canada

Applications of Quantum cohomology to some fundamental problems of dynamics

I will first present briefly the origins and the construction of Quantum cohomology, and I will then indicate how the fundamental group of Hamiltonian diffeomorphisms acts on that cohomology. This leads to new invariants attached to groups of diffeomorphisms, and has immediate applications to the topology of Hamiltonian dynamical systems.

YURIY SHKOLNIKOV, Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta T2N 1N4, Canada

A generalisation of Whitney's trick in dimension 4, borromeanism and related questions

It is well known that the classical Whitney's trick for 2-submanifolds X and Y of a 1-connected PL (or smooth) 4-manifold M does not work. It has not been noticed that the obstruction to it lies in the topology of Borromean-like links in S^3. This gives a rise to an idea of simultaneous performing of the Whitney-like trick with 2 or more Whitney pairs of intersection points of X and Y. Under certain conditions such a trick (called a Whitney's multitrick) turns out to be successful. The distinctive feature of a multitrick is that it can be performed only collectively which means that for each separate pair of Whitney's points the classical trick does not work.

JENNIFER SLIMOWITZ, Université du Québec à Montréal, Montréal, Québec H3C 3P8, Canada

Length minimizing geodesics in the group of Hamiltonian diffeomorphisms

To any symplectic manifold (M, ω), one can associate the group $\mathrm{Ham}^c(M)$ of compactly supported Hamiltonian diffeomorphisms of M. Hofer has constructed a norm on this group which can be used to define the notion of a length minimizing path. A new class of examples of length minimizing paths in $\mathrm{Ham}^c(M)$ for M of dimension 2 or 4 will be presented. The proofs rely on a technique described by Lalonde and McDuff using a new estimate on the Hofer-Zenhder capacity of certain manifolds.

XINGRU ZHANG, Mathematics Department, SUNY at Buffalo, Buffalo, New York 14214-3093, USA

On simple points of the character variety of a cusped hyperbolic 3-manifold

In recent years, the study of the $\mathrm{SL}(2, C)$-character varieties of 3-manifolds has brought great progress in underlying the topology and geometry of 3-manifolds. Yet many fundamental questions concerning these varieties remain unanswered. In this talk I will discuss one aspect of the character variety of a cusped hyperbolic 3-manifold, namely to determine which points of the variety are simple in the sense of algebraic geometry. This is a joint work with Steve Boyer.

Universal Algebra and Multiple-Valued Logic / Algèbre universelle et logique multi-valuée

L. Haddad, Organizer

CLIFFORD BERGMAN, Department of Mathematics, Iowa State University, Ames, Iowa 50011, USA

Complexity of some problems in universal algebra

Given two finite algebras A and B, there are several natural questions one might ask about the relationship between A and B. Among them: Do A and B generate the same (quasi)variety? And, are A and B term-equivalent? Although these problems are well-known to be decidable, the precise computational complexity seems to be nontrivial.

In this talk, we will survey some old and new results on these questions, as well as pose some open problems.

This is joint work with Giora Slutzki (Iowa State University).

JIE FANG, Department of Mathematics, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada

Ockham algebras with pseudocomplementation

The variety $\mathbf{pO}$ consists of those algebras $(L, \wedge, \vee, f, \star, 0, 1)$ of type $\langle 2, 2, 1, 1, 0, 0\rangle$, where $(L, \wedge, \vee, f, 0, 1)$ is an Ockham algebra, $(L, \wedge, \vee, \star, 0, 1)$ is a p-algebra, and the unary operations f and $\star$ commute. We describe the structure of the subdirectly irreducible algebras that belong to the subclass $\mathbf{pK}_{1,1}$ characterised by the property $f^3 = f$, and give a description of the lattice of subvarieties of $\mathbf{pK}_{1,1}$.

IBRAHIM GARRO, Institute for the History and Philosophy of Science and Technology, University of Toronto, Toronto, Ontario M5S 3G3, Canada

An application of non-wellfounded sets to infinite valued infinitary propositional calculus

The present article is a continuation of my work [1] demonstrating natural applications of non-wellfounded sets in logic.

A stream is given as a domain of interpretation for infinitary propositional logic. This stream is the set of ordered pairs $V = \Big(0, \big(1, (2, \ldots)\big)\Big)$. 0 is the empty set and the second component of every pair is taken as its complement in V.

The logical operations are interpreted in V in the manner of post algebras. Negation is interpreted as complement. 'or' is interpreted as max. and 'and' as min. (with special care when one of the terms is infinite expression *i.e.*, a negation, and for infinitely long expressions.)

We then show that we get as a special case the generalized post algebra Pw described in Urquhart [2] p. 91. This opens the road to applications of the methods used in Post algebras to streams and vice-versa.

We then study the extension of these results to predicate logic and look at its implications on the complexity of the decision problem for propositional logic by comparing the underlying set theories.

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GEORGE GRÄTZER, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

Independence Theorems for automorphism groups and congruence lattices of lattices

In my book, *General Lattice Theory*, in 1978, I raised the following problem:

Let L be a lattice and let G be a group. Does there exist a lattice K such that K and L have isomorphic congruence lattices and the automorphism group of K is isomorphic to G?

Several papers have been published providing technical tools that could be used to attack this problem, in particular,

- earlier papers by G. Birkhoff and R. Frucht;
- more recent papers by V. A. Baranskiĭ, M. Tischendorf, and A. Urquhart;
- a number of relevant categorical results by the Prague group (reported, in part, in a book of A. Pultr and V. Trnková);
- a number of papers by G. Grätzer and E. T. Schmidt on congruence-preserving extensions;
- earlier papers on the topic of tensor products of lattices with zero by J. Anderson and N. Kimura, G. A. Fraser, and G. Grätzer, H. Lakser, and R. W. Quackenbush and a series of very recent papers on the same topic (and on some generalizations) by G. Grätzer and F. Wehrung.

Based on these contributions, F. Wehrung and I have succeeded in solving this problem.

To state the new results, we need two definitions.

Let L be a lattice. A lattice K is a *congruence-preserving extension* of L, if K is an extension of L and every congruence of L extends to exactly one congruence of K. Of course, then the congruence lattice of L is isomorphic to the congruence lattice of K.

A lattice K is an *automorphism-preserving extension* of L, if K is an extension of L and every automorphism of L has exactly one extension to K, and in addition, every automorphism of K is the extension of an automorphism of L. Of course, then the automorphism group of L is isomorphic to the automorphism group of K.

The Strong Independence Theorem for Lattices with Zero.

Let L_{A} and L_{C} be lattices with zero, let L_{C} have more than one element. Then there exists a lattice K that is a $\{0\}$-preserving extension of both L_{A} and L_{C}, an automorphism-preserving extension of L_{A}, and a congruence-preserving extension of L_{C}.

The Strong Independence Theorem for Lattices.

Let L_{A} and L_{C} be lattices, let L_{C} have more than one element. Then there exists a lattice K that is an automorphism-preserving extension of L_{A} and a congruence-preserving extension of L_{C}.

JENNIFER HYNDMAN, Mathematics and Computer Science, University of Northern British Columbia, Prince George, British Columbia V2N 4Z9, Canada

Dualizable is not the same as fully dualizable

I will present a bi-unary algebra that is dualizable but not fully dualizable in the sense of natural duality. In fact the algebra is not fully dualizable by any alter ego with only a set of relations, operations or partial operations in its signature. This is joint work with R. Willard (University of Waterloo).

BENOIT LAROSE, Champlain Regional College, Quebec, Canada
Projective graphs and Hedetnyemi's conjecture

Let $\mathbf{G}$ be a finite simple graph, which for our purposes we define as a pair $\mathbf{G} = \langle \mathbf{G}, \theta \rangle$ where θ is a symmetric and areflexive binary relation on the finite set G. We say that $\mathbf{G}$ is *n-projective* if the only n-ary idempotent operations in the clone Polθ are the projections, and $\mathbf{G}$ is *n-quasiprojective* if all the n-ary idempotent operations in the clone Polθ preserve every subset of G. We prove that for all $n \geq 2$, n-projectivity, n-quasiprojectivity and 2-quasiprojectivity are equivalent. (the result also holds for infinite graphs). This can be used to show that complete graphs, odd cycles and Kneser graphs are n-projective for every $n \geq 2$.

These results are related to Hedetnyemi's conjecture, which states that the chromatic number of the product of two graphs is the minimum of the chromatic numbers of the factors; indeed, this conjecture can be reformulated as follows: every complete graph K_n is *multiplicative*, *i.e.* if K_n is a retract of a product $G \times H$ then it is a retract of G or H. We show that under certain conditions, a projective graph $\mathbf{P}$ is *weakly multiplicative*, *i.e.* that if $\mathbf{P}$ is a retract of a product of *connected* graphs then it is a retract of one of the factors. In particular, complete graphs and Kneser graphs are shown to be weakly multiplicative. (Note that there exist Kneser graphs which are not multiplicative (Tardif, Zhou)).

This is joint work with C. Tardif.

JONATHAN LEECH, Department of Mathematics, Westmont College, Santa Barbara, California 93108-1099, USA
Noncommutative lattices: foundational issues and recent results

Since the 1940s, a number of individuals have considered algebras which, while noncommutative, nonetheless resemble lattices. Typically, such algebras include a pair of binary operations which are assumed to be associative and to satisfy certain absorption identities, but are not necessarily commutative. Because of the different types of absorption possible in a noncommutative context, there has been considerable variation in the classes of algebras introduced. The talk will focus on issues such as: What should one expect a noncommutative lattice to look like? What is the role of lattice theory in framing such an expectation? Likewise, what is the role of semigroup theory? These and other issues will be discussed in the light of recent developments, some as yet unpublished.

ROBERT W. QUACKENBUSH, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada
Varieties of binary linear codes

A binary linear code is a vector space V over GF(2) with an added unary operation, $'$, satisfying $0' = 0$; $x'' = x'$, and $(x' + y)' = x' + y'$. This last law exactly expresses the facts that the set of codewords $C := \{x' \mid x \in V\}$ is a subspace and that $'$ on any coset of C is a translation by a fixed element. I will discuss the lattice of subvarieties and discuss some connections to classical linear codes, *e.g.*, Hamming codes are closely related to the n-generated free codes in the variety.

I. G. ROSENBERG, Département de mathématiques et statistique, Université de Montréal, Montréal, Quebec H3C 3J7, Canada
Completeness for uniformly delayed circuits

We survey algebraic results on combinatorial circuits constructed from many-valued uniformly delayed gates. These involve the composition ofniformly delayed operations, the lattice of uniform clones, the corresponding relational theory, uniform completeness and the search for an effective completeness criterion. This is joint work with T. Hikita, Dept of Computer Science, Meiji University, Japan.

ROSS WILLARD, Pure Mathematics Department, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
Independence of the linear commutator

Years ago, Lampe discovered some unexpected interactions between the congruence lattice of an algebra (as an abstract lattice) and the term-condition commutator operation on this lattice. We claim that no result of this kind is possible for the linear commutator. In particular, no lattice-theoretic property of the congruence lattice of an algebra can force the algebra to be quasi-affine. This is joint work with K. A. Kearnes (University of Louisville) and W. A. Lampe (University of Hawaii).

Education I/ Éducation I
Identifying and Overcoming Barriers to Teaching and Learning Mathematics at University / Identifier et vaincre les obstacles à l'enseignement et l'apprentissage des mathématiques à l'université

Morris and Grace Orzech, Organizers

EDWARD BARBEAU, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 3G3, Canada
The teacher as coach

Effective learning by students depends in part on their perception of the nature of mathematics and how it is to be mastered, as well as on their understanding of their goals, abilities and learning styles. Such issues need to be raised explicitly to avoid a conflict of expectation between student and teacher and to allow students to marshall their resources most effectively. This talk will discuss how failure to consider such factors can impede progress.

BILL BEYERS, Department of Mathematics and Statistics, Concordia University, Montreal, Quebec H4B 1R6, Canada
Revealing the inner mathematician

Teachers of university level mathematics often present the subject to their students in a way which is very different from the way they themselves understand it. The student is expected to see through the surface of the presentation (often rigorous, axiomatic) and somehow "get it". How you go about "getting it" and what it is precisely that you are supposed to "get" is usually unclear to the student.

Part of the problem is the logical structure of mathematics. When should rigor be introduced? In what context? And to which students? Bill Thurston wrote that "what we [should be]...doing [as mathematicians and teachers] is finding ways for *people* to understand and think about mathematics". As teachers of mathematics, we must show the students how to think by revealing honestly how we think about mathematics.

These questions will be discussed in the context of various curricular reforms that we have undertaken at Concordia.

MORRIS ORZECH, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6, Canada
Addressing student difficulties specific to linear algebra

At my university, first year students who plan a mathematics concentration as part of their degree programme take courses in linear algebra and in calculus. Each course offers its challenges, but linear algebra seems a more uncomfortable experience for both teachers and students. My outlook on this phenomenon is that linear algebra seems to be more difficult for students despite it being taught with as much care, and as much attention to content, as is calculus. This viewpoint has led me to shift my attention from an almost exclusive focus on choosing material and presenting it clearly to an increased concern with aligning the way material is offered to the students' abilities to learn it. Some of my attempts in this direction depart from my traditional teaching practice by looking for guidance to learning-theory based ideas of people like Harel, Hillel and Sierpinska. Some are more ad-hoc, but are unconventional in other ways: in the name of having students better learn and appreciate mathematics they challenge some shibboleths that our conventional teaching practice seeks to inculcate.

THOMAS W. RISHEL, Cornell University, Ithaca, New York 14853-7901, USA
Teaching and job initiatives for graduate programs in mathematics

In the past few years, a number of new initiatives have developed for preparation of graduate students for the professoriate. Among these are TA training programs, college teaching courses, "professors for the future" programs, job search and placement services, peer mentoring, and teaching portfolios.

Rishel, who has been involved in many of the above for the mathematics department at Cornell, will outline the design, implementation and rationale for such initiatives.. He will also provide sample materials from his program.

MARTHA SIEGEL, Towson University, Towson, Maryland 21252-7097, USA
To be announced

KEITH F. TAYLOR, University of Saskatchewan, Saskatoon, Saskatchewan S7N 5E6, Canada
The Math readiness project at the University of Saskatchewan

For several years, we have been developing and conducting a preuniversity preparation programme in mathematics at the University of Saskatchewan. Among the components in the programme are a two week summer camp of intensive review, evening versions of the course, a distance delivery version that uses the internet, and an internet based course on basic mathematics for mature students who have forgotten everything. I will describe all aspects of Math Readiness including our successes and failures and how we fund the project. If we have the right technology available, I will give a demonstration of MRC (the Math Readiness Course on the internet).

Education II/ Éducation II
Teaching Mathematics—Why We do What We do in the Classroom? / L'enseignement des mathématiques—pourquoi fait-on ce qu'on fait en classe?

William Higginson, Grace and Morris Orzech, Organizers

GEORGE GADANIDIS, Durham District School Board, 400 Taunton Road East, Whitby, Ontario L1R 2K6, Canada
If Piaget was a math teacher—Theoretical images, classroom practice and the power of students' minds

This talk will address theory-practice links at the classroom level. Subject to the availability of time and equipment we will:

- discuss the theory-practice gap/overlap at the classroom level;
- illustrate with video clips how theoretical images may be translated into practice;
- discuss how we may work with teachers so that they develop and experiment with personal theoretical images.

WILLIAM HIGGINSON, Faculty of Education, Queen's University, Kingston, Ontario K7L 3N6, Canada
Having, knowing, and being: some fundamental questions about mathematics teaching

[Abstract not available]

LYNN MCALPINE AND CYNTHIA WESTON, Centre for University Teaching and Learning, McGill University, Montreal, Quebec H3A 2K6, Canada
How six outstanding math professors use reflection to improve their teaching

An increased value is being placed on quality teaching in higher education. An important step in developing approaches to better instruction is understanding how those who are successful at teaching go about improving their own teaching. Although reflection has frequently been referred to as a mechanism for accomplishing this improvement process (in both literatures on university and public school teaching), there had been no research which documented it as a cognitive process. So, a number of years ago, we undertook a program of research in which the concept of "reflection" provided the frame of reference. We wanted to see if and how it improved practice.

We chose to look at outstanding professors since the literature suggests that experts tend to reflect more. And, we chose mathematics as the subject area since there appears to be a general public concern about the quality of students' learning in this area. We investigated in detail the reflective processes of six professors each teaching an undergraduate course in mathematics or mathematics education. We tracked their day-to-day planning, instructing and evaluating of learners. We documented what these professors were monitoring during and after teaching, what changes they made as a result, and what knowledge they drew upon in making decisions. Of particular interest was their attention to feedback from students in making decisions about how to adjust their teaching.

What we will do in the presentation is describe how these professors used reflection to improve their teaching, and present the model of reflection that resulted from the analysis. We have found that this model provides a language for describing reflection and therefore a way to be intentional in using reflection.

PAT ROGERS, Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada
The importance of why: encouraging students to reflect

[Abstract not available]

NATHALIE SINCLAIR, Queen's University, Kingston, Ontario K7L 3N6, Canada
Romancing powerful mathematical ideas

Technology can provide students with interactive tools for visualizing, transforming and simulating mathematical objects; this enables them to explore powerful mathematical ideas, find connections between them and construct their knowledge of them by creating their own microworlds. Classroom experiences in trying to achieve this potential with middle school students will be presented, as well as a description of the JavaBean-based technologies developed and used.

Mathematics on the Internet / Mathématiques sur internet

June Lester, Nathalie Sinclair and Malgorzata Dubiel, Organizers

WILLIAM CASSELMAN, Department of Mathematics, University of British Columbia, Vancouver, British Columbia V6T 1Z2, Canada

Colour, animation, interaction—the next generation of electronic journals

So far, electronic journals have only acted as cheaper versions of paper journals. In the near future we can expect to see more imaginative productions, but they will require much more work by both authors and staff. How can we prepare for this? What will be the rewards?

STAN DEVITT, Waterloo Maple Inc., Waterloo, Ontario, Canada

Extracting mathematical meaning from MathML notation—an essential step towards live math on the Web

The meaningful representation of mathematics on the web has had a checkered past. Approaches have fallen into essentially three categories: Pictures of formulae, prepared using other software such as Maple, Expressionist, TeX or Mathtype, embedded special encodings processed by plugins and applets such as techexplorer or WebEQ, and pictures of entire pages formatted using page layout software such as PageMaker, TEXor FrameMaker.

While there are extensive problems deriving from scaling and alignment of the pictorial fragments, especially after rescaling of the page, each of these approaches has a much more fundamental flaw. Their heavy reliance on pictures of formulae and page layout based encodings prevents the encoding the mathematical meaning of the original formulae. Without a mechanism for the author to specify the intended semantics, there no way to automatically process formulae that are embedded on web pages.

W3C's MathML 1.0 specification addresses the semantics issues by providing encodings for both the visual/aural encoding and the mathematical semantics. The approach taken allows authors to actually define and reference their notation. This talk explores how MathML allows you to work with the mathematical meaning of expressions, especially when interacting with computer algebra systems such as Maple.

STAN DEVITT, Waterloo Maple Inc., Waterloo, Ontario, Canada

The Relationship between OpenMath and MathML

The primary purpose of OpenMath is to facilitate reliable communication of mathematical objects between mathematical applications. To accomplish this the actual meanings of the objects must be available. To see why consider that in the absence of such definitions we would be unable to determine if D^2y represents two applications of the differential operator D to the function y, or is simply a monomial corresponding to the square of the symbol D multiplied by the the symbol y, where D and y are elements in some algebra chosen by the author. In this talk we review the current status of the OpenMath standard, how it is being used, and how this relates to similar requirements for the communication of mathematical meaning in MathML.

NICHOLAS JACKIW, KCP Technologies, 1150 65th Street, Emeryville, California 94608, USA
Taking dynamic geometry on-line

This session will investigate educational and technical issues surrounding JavaSketchpad, a prototype web-based version of The Geometer's Sketchpad which emerged from research into pen-based mathematical modeling software. The software and some examples of its use shall be demonstrated, and the author will discuss both the specific technical difficulties and mathematical questions posed by its "dynamic geometry" paradigm; and more generally, the Internet's current position within the technology landscape of secondary mathematics education.

LOKI JÖRGENSON, Centre for Experimental and Constructive Mathematics, Simon Fraser University, 8888 University Drive, Burnaby Mountain Campus, Burnaby, British Columbia V5A 1S6, Canada
The CMS Journals On-Line: A Study in Digital Publishing

A guided tour of the Canadian Journal of Mathematics and the Canadian Mathematics Bulletin On-line will be presented with an opportunity for the audience to ask questions and make suggestions for their future. Plans for their development will be sketched out with reference to some of the new Web technologies presented at the conference. Predictions for the future of digital publishing in mathematics will be offered, some disquieting, some controversial and others exciting, but all interesting to those concerned about scholarly communications.

ROBERT MINER, Geometry Technologies, Inc., 400 Sibley Street, St. Paul, Minnesota 55101, USA
Putting math on the Web

Nearly eight years after the invention of the World Wide Web, it is still a difficult medium to use for the communication of mathematical and scientific material in spite of its phenomenal success in other areas. In this talk, we will briefly survey the status of current technologies for putting math on the Web, and in particular the development and implementation of the World Wide Web Consortium's Mathematical Markup Language (MathML).

Contributed Papers Session / Communications libres

L. Haddad, Organizer

ALEXANDRU GICA, Faculty of Mathematics, University of Bucharest, RO-70109 Bucharest 1, Romania

A conjecture which implies the theorem of Gauss-Heegner-Stark-Baker

In 1801 Gauss conjectured that the ring of integers for a quadratic imaginary field $K = \mathbf{Q}(\sqrt{d})$ is principal only for a finite number of d. Heegner, Stark and Baker proved that only for

$$-d = 1, 2, 3, 7, 11, 19, 43, 67, 163$$

the ring of integers for $\mathbf{Q}(\sqrt{d})$ (where $d \in \mathbf{Z}$, $d < 0$, d squarefree) is principal. We prove two result which are equivalent with this theorem and we pose a conjecture which implies the Gauss-Heegner-Stark-Baker theorem (we will abbreviate: the G.H.S.B. theorem). Finally we pose a "weaker" conjecture which seems to be more approachable than the other one.

I. NIKOLAEV, CRM, Université de Montréal, Montréal, Québec H3C 3J7, Canada

3-manifolds, foliations and $K_0(C(X) \rtimes Z)$

J. F. Plante [1] classified 3-dimensional manifolds M such that every foliation $\mathcal{F}$ on M has a compact leaf. Plante's method is based on estimation of 'growth rate' of the fundamental group $\pi_1(M)$.

We show that calculation of a 'dimension function' of K_0-group of the crossed product C^*-algebra generated by $\mathcal{F}$ leads to series of results covering Plante's classification.

References

1. J. F. Plante, *Foliations with measure preserving holonomy.* Ann. of Math. **102**(1975), 327–361.

DIETER RUOFF, Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan S4S 0A2, Canada

Solution of a non-Euclidean convexity problem

The curve that will be investigated is made up from the vertices of the angles which have size α, share the common chord AB and lie in one and the same halfplane with respect to the line through AB. It is a well-known fact that in Euclidean geometry these points form an arc of a circle. Also, it is clear that any one of the given angles can be moved into any other by a rotation. In non-Euclidean geometry all this does not hold. A sketch of a convexity proof for this latter setting will be provided.

ANDREW TOMS, Fields Institute, University of Toronto, Toronto, Ontario, Canada

Perforated K_0-*groups of* C^*-*algebras*

Every weakly unperforated ordered abelian group arises as the K_0−group of some simple, separable C^*−algebra. What of perforated abelian groups? In this talk we outline methods for constructing C^*−algebras having a maximal free cyclic subgroup of K_0, say H, with the property that $H \cap K_0^+ \simeq \langle k, l \rangle$, k and l relatively prime positive integers. The methods used also provide many other such algebras with subgroups whose intersection with the positive cone of K_0 further extend the preceding result, though these are less succinctly described.

Graduate Student Seminar / Séminaire pour étudiants diplômés

David Gregory, Organizer

LEO BUTLER, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6, Canada.

A New Class of Homogeneous Manifolds with Liouville-Integrable Geodesic Flows

A family of nilmanifolds possessing Riemannian metrics whose geodesic flow is Liouville-integrable is demonstrated. These homogeneous spaces are of the form $D\backslash H$, where H is a connected, simply-connected and two-step nilpotent Lie group and D is a discrete, cocompact subgroup of H. The metric on these homogeneous spaces is obtained from a left-invariant metric on H. These nilmanifolds provide the first example of manifolds whose fundamental group possesses no commutative subgroup of finite index, yet they admit a Liouville-integrable geodesic flow. The conclusions of Taĭmanov's theorem do not obtain in the category of Liouville-integrable geodesic flows with smooth first integrals.

LEO CREEDON, Department of Mathematical Sciences, University of Alberta, Edmonton, Alberta T6G 2G1, Canada

Constructing free groups in modular group algebras

We investigate the existence (and construction) of free pairs of units in the unit group of a (modular) group algebra KG. We generalise a result of Goncalves/Passman to do this, and use the programming package GAP to investigate the units of $\mathbb{F}_2(t)A_5$, where t is an element transcendental over $\mathbb{F}_2$.

MARK V. DEFAZIO, Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada

The behaviour of the complex zeroes of the Laguerre polynomial

When $\alpha > -1$, the behaviour of the zeroes of $L_n^{(\alpha)}(x)$ is well understood. There are n positive distinct real zeroes and they are continuously increasing functions of α. When $\alpha < -1$, these zeroes can be complex and their behaviour as α varies is not well known. I will present results on the location of the complex zeroes of $L_n^{(\alpha)}(z)$ and describe their behaviour as $\alpha < -1$ varies.

JODY ESMONDE, Department of Mathematics and Statistics, McGill University, Montréal, Québec H3A 2K6, Canada

Parametric solutions to the generalized Fermat equation

We will discuss parametric solutions $a(t), b(t), c(t) \in F[t]$ to the equation $x^p + y^q = z^r$, proving that such solutions exist if and only if $1/p + 1/q + 1/r > 1$.

SHAUN FALLAT, Department of Mathematics, College of William and Mary, Williamsburg, Virginia 23187, USA

Multiplicative principal minor inequalities for totally nonnegative matrices

An m-by-n matrix A is said to be *totally nonnegative* if the determinant of every square submatrix (*i.e.*, *minor*) of A is nonnegative. Our main interest lay in characterizing all the inequalities that exist among products of principal minors (minors of A based on the same row and column index sets) of nonsingular totally nonnegative matrices. We give a complete characterization of all such inequalities for n-by-n totally nonnegative matrices with $n \leq 5$. We also illustrate certain key steps in the proof of this characterization, which offer interesting applications of symbolic computation, graph theory, convexity, and linear programming. Other results are presented including general conditions which guarantee when the product of two principal minors is less the product of two other principal minors, and many others.

This is joint work with Professors M. Gekhtman and C. Johnson.

ANDREI V. GAGARIN, Department of Mathematics, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada

Characterizations of (α, β)-polar graphs by forbidden induced subgraphs

Let α and β be positive integers or infinity. A graph G is called (α, β)-*polar* if there is a partition of its vertex set $V(G) = A \cup B$, $A \cap B = \varnothing$, such that the induced subgraph $G(B)$ is a union of disjoint cliques of order at most β and $G(A)$ is the complement to a union of disjoint cliques of order at most α.

In this talk, we will consider the problem of characterizing (α, β)-polar classes in terms of a finite list of forbidden induced subgraphs.

MALCOLM HARPER, Department of Mathematics and Statistics, McGill University, Montreal, Quebec H3A 2K6, Canada

A family of Euclidean rings containing $Z[\sqrt{14}]$

Let K be an algebraic number field with ring of integers $\mathcal{O}_K$ and suppose that $\mathcal{O}_K$ has an infinite unit group. Assumming a suitable generalized Riemann hypothesis, $\mathcal{O}_K$ is a Euclidean ring (in the sense of Samuel, 1971) if and only if K has class number 1 (Weinberger, 1973). $Z[\sqrt{14}]$ has an infinite unit group and is the ring of integers in $K = Q(\sqrt{14})$ which has class number 1. Cardon (1997) showed that the fundamental obstruction to the norm acting as a Euclidean algorithm in $Z[\sqrt{14}]$ lies at one of the residue classes modulo 2 and thus $Z[\sqrt{14}][1/2]$ is Euclidean. Using the sieve techniques of Gupta, Murty and Murty (1987), Clark (1992) and Clark and Murty (1995) we show that inverting any non-unit in $Z[\sqrt{14}]$ yields a Euclidean ring. That is, $Z[\sqrt{14}][1/a]$ is Euclidean for any a in $Z[\sqrt{14}]$ not a unit.

YU-RU LIU, Department of Mathematics, Harvard University, Cambridge, Massachusetts 02138-2901, USA

The Turán sieve and probabilistic Galois theory

We introduce the Turán sieve and apply it to the probabilistic Galois theory problem in both the rational and the functional cases. More precisely, we estimate the number of polynomials of degree n and height $\leq N$ whose Galois group is a proper subgroup of S_n.

DAVID MCKINNON, Department of Mathematics, University of California, Berkeley, California 94618, USA

An arithmetic Bézout theorem

In this paper, we prove two versions of an arithmetic analogue of Bézout's theorem, subject to some technical restrictions. The basic formula proven is $h(X \cap Y) = h(X)\deg(Y) + h(Y)\deg(X) + O(1)$. The theorems are inspired by the arithmetic Bézout theorem of Bost, Gillet, and Soulé, but improve upon them in two ways. First, we obtain an equality up to $O(1)$ as the intersecting cycles vary in projective families. Second, we generalise this result to intersections of divisors on any regular, generically smooth, projective arithmetic variety. We present an application of these results to proving an analogue of Hilbert's Irreducibility Theorem for intersections of curves in $\mathbf{P}^2$.

SATYA MOHIT, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6, Canada
The ABC-Conjecture and bounds for the order of the Tate-Shafarevich group

We will discuss various equivalent formulations of the ABC-Conjecture in terms of bounds for the order of invariant quantities associated to elliptic curves, particularly, the Tate-Shafarevich group. We will show that, in order to prove the ABC-Conjecture, it suffices to prove the following fact:

For every elliptic curve E, there is some 'small' quadratic twist E_D whose Tate-Shafarevich group is bounded by $DN^{1/2+\epsilon}$.

DANIEL PICHÉ, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
Wavelet compression on fractal tilings

Over the last decade, wavelets have become increasingly useful for studying the behaviour of functions, and for compression. Though much remains to be investigated in this field, certain types of wavelets are fairly easy to construct, namely Haar wavelets. This paper ties together the theory of these wavelets with that of complex bases. An algorithm is proposed, for doing wavelet analysis, with wavelets arising in this fashion. This will enable further study of the properties of these wavelets.

FILIP SAIDAK, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6, Canada
On zero-free regions of the zeta function

A powerful analytic method of Turán brought several advances in understanding the local behaviour of zeros of zeta-like functions. We discuss certain simplifications and extensions of these classical results, highlighting the main unsolved problems and conjectures of the theory.

GREGORY S. SMITH, Department of Mathematics, University of California at Berkeley, Berkeley, California 94720-3840, USA
Computing global extension modules

Let X be a projective scheme; let $\mathcal{M}$ and $\mathcal{N}$ be two coherent $\mathcal{O}_X$-modules. Given an integer m, we present an algorithm for computing the global extension module $\mathrm{Ext}^m(X\ ;\mathcal{M},\mathcal{N})$. In particular, this allows one to calculate the sheaf cohomology $H^m(X,\mathcal{N})$ and to construct the sheaf corresponding to an element of the module $\mathrm{Ext}^1(X\ ;\mathcal{M},\mathcal{N})$. This algorithm can be implemented using only the computation of Gröbner bases and syzygies, and it has been implemented in the computer algebra system *Macaulay2*.

SARAH SUMNER, Department of Mathematics and Statistics, Queen's University, Kingston, Ontario K7L 3N6, Canada
Investigating transcendence in the field of p-adic numbers

In the field of p-adic numbers, whether the number $\sum_{n=1}^{\infty} n!$ is transcendental or even irrational is an unsolved problem. However, it can be shown that numbers of the form $\sum_{n=1}^{\infty} n^k \cdot n!$ can be written in the form $v_k - u_k \sum_{n=1}^{\infty} n!$ where u_k and v_k are integers. Interesting congruences involving the series $\{u_k\}$ are developed and conditions that will produce a p-adic transcendental number are discussed.

This is joint work with Professor Ram Murty.

DREW VANDETH, Macquarie University, Centre for Number Theory Research, Sydney, Australia
Transcendence of the values of generalized Mahler functions

In this talk we examine conditions under which generalized Mahler functions, of one or several variables, assume transcendental values at given algebraic points. The results are achieved through the injection of the Bombieri-Vaaler Siegel Lemma into the work of P-G. Becker and the work of F. Gramain, M. Mignotte and M. Waldschmidt.