PROBLEMS FOR JUNE

Please send your solution to

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no later than July 31, 2007. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

500. Find all sets of distinct integers $1 < a < b < c < d$ for which $abcd - 1$ is divisible by $(a-1)(b-1)(c-1)(d-1)$.

501. Given a list of $3n$ not necessarily distinct elements of a set S, determine necessary and sufficient conditions under which these $3n$ elements can be divided into n triples, none of which consist of three distinct elements.

502. A set consisting of n men and n women is partitioned at random into n disjoint pairs of people. What are the expected value and variance of the number of male-female couples that result? (The *expected value* E is the average of the number N of male-female couples over all possibilities, *i.e.* the sum of the numbers of male-female couples for the possibilities divided by the number of possibilities. The *variance* is the average of the difference $(E-N)^2$ over all possibilities, *i.e.* the sum of the values of $(E-N)^2$ for the possibilities divided by the number of possibilities.)

503. A natural number is *perfect* if it is the sum of its proper positive divisors. Prove that no two consecutive numbers can both be perfect.

504. Find all functions f taking the real numbers into the real numbers for which the following conditions hold simultaneously:

(a) $f(x + f(y) + yf(x)) = y + f(x) + xf(y)$ for every real pair (x, y);

(b) $\{f(x)/x : x \neq 0\}$ is a finite set.

505. What is the largest cubical present that can be completely wrapped (without cutting) by a unit square of wrapping paper?

506. A two-person game is played as follows. A position consists of a pair (a, b) of positive integers. Playes move alternately. A move consists of decreasing the larger number in the current position by any positive multiple of the smaller number, as long as the result remains positive. The first player unable to make a move loses. (This happens, for example, when $a = b$.) Determine those positions (a, b) from which the first player can guarantee a win with optimal play.