

Problems for February, 2006

Please send your solution to

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no later than March 28, 2006. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

Notes: Given a triangle, extend two nonadjacent sides. The circle tangent to these two sides and to the third side of the triangle is called an *excircle*, or sometimes an *escribed circle*. The centre of the circle is called the *excentre* and lies on the angle bisector of the opposite angle and the bisectors of the external angles formed by the extended sides with the third side. Every triangle has three excircles along with their excentres.

The *incircle* of a polygon is a circle inscribed inside of the polygon that is tangent to all of the sides of a polygon. While every triangle has an incircle, this is not true of all polygons.

430. Let triangle ABC be such that its excircle tangent to the segment AB is also tangent to the circle whose diameter is the segment BC . If the lengths of the sides BC , CA and AB of the triangle form, in this order, an arithmetic sequence, find the measure of the angle ACB .
431. Prove the following trigonometric identity, for any natural number n :

$$\sin \frac{\pi}{4n+2} \cdot \sin \frac{3\pi}{4n+2} \cdot \sin \frac{5\pi}{4n+2} \cdots \sin \frac{(2n-1)\pi}{4n+2} = \frac{1}{2^n}.$$

432. Find the exact value of:

(a)

$$\sqrt{\frac{1}{6} + \frac{\sqrt{5}}{18}} - \sqrt{\frac{1}{6} - \frac{\sqrt{5}}{18}};$$

(b)

$$\sqrt{1 + \frac{2}{5}} \cdot \sqrt{1 + \frac{2}{6}} \cdot \sqrt{1 + \frac{2}{7}} \cdot \sqrt{1 + \frac{2}{8}} \cdots \sqrt{1 + \frac{2}{57}} \cdot \sqrt{1 + \frac{2}{58}}.$$

433. Prove that the equation

$$x^2 + 2y^2 + 98z^2 = 77777 \dots 777$$

does not have a solution in integers, where the right side has 2006 digits, all equal to 7.

434. Find all natural numbers n for which $2^n + n^{2004}$ is equal to a prime number.
435. A circle with centre I is the incircle of the convex quadrilateral $ABCD$. The diagonals AC and BD intersect at the point E . Prove that, if the midpoints of the segments AD , BC and IE are collinear, then $AB = CD$.
436. In the Euro-African volleyball tournament, there were nine more teams participating from Europe than from Africa. In total, the European won nine times as many points as were won by all of the African teams. In this tournament, each team played exactly once against each other team; there were no ties; the winner of a game gets 1 point, the loser 0. What is the greatest possible score of the best African team?