Corrected Problems for October, 2005

There are a couple of corrections and clarifications for the October set of problems. The most important is to note that Dr. Yen's address is
7255 Hewitt Street

The corrected version of the problem set is given below. Amendments have been made to Problems 409, 412, 414:

Please send your solutions to
Dr. Lily Yen
7255 Hewitt Street
Burnaby, BC V5A 3M3

no later than November 30, 2005 It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

409. Find the number of ways of dealing n cards to two persons ($n \geq 2$), where the persons may receive unequal (positive) numbers of cards. Disregard the order in which the cards are received.

410. Prove that $\log n \geq k \log 2$, where n is a natural number and k the number of distinct primes that divide n.

411. Let b be a positive integer. How many integers are there, each of which, when expressed to base b, is equal to the sum of the squares of its digits?

412. Let A and B be the midpoints of the sides, EF and ED, of an equilateral triangle DEF. Extend AB to meet the circumcircle of triangle DEF at C. Show that B divides AC according to the golden section. (That is, show that $BC : AB = AB : AC$.)

413. Let I be the incentre of triangle ABC. Let A', B' and C' denote the intersections of AI, BI and CI, respectively, with the incircle of triangle ABC. Continue the process by defining I' (the incentre of triangle $A'B'C'$), then $A''B''C''$, *etc.*. Prove that the angles of triangle $A^{(n)}B^{(n)}C^{(n)}$ get closer and closer to $\pi/3$ as n increases.

414. Let $f(n)$ be the greatest common divisor of the set of numbers of the form $k^n - k$, where $2 \leq k$, for $n \geq 2$. Evaluate $f(n)$. In particular, show that $f(2n) = 2$ for each integer n.

415. Prove that

$$\cos\frac{\pi}{7} = \frac{1}{6} + \frac{\sqrt{7}}{6}\left(\cos\left(\frac{1}{3}\arccos\frac{1}{2\sqrt{7}}\right) + \sqrt{3}\sin\left(\frac{1}{3}\arccos\frac{1}{2\sqrt{7}}\right)\right).$$