PROBLEMS FOR JUNE-JULY

Please send your solution to
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no later than July 31, 2005. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

388. A class with at least 35 students goes on a cruise. Seven small boats are hired, each capable of carrying 300 kilograms. The combined weight of the class is 1800 kilograms. It is determined that any group of 35 students can fit into the boats without exceeding the capacity of any one of them. Prove that it is unnecessary to leave any student off the cruise.

389. Let each of m distinct points on the positive part of the $x-$axis be joined by line segments to n distinct points on the positive part of the $y-$axis. Obtain a formula for the number of intersections of these segments (exclusive of endpoints), assuming that no three of the segments are concurrent.

390. Suppose that $n \geq 2$ and that $x_1, x_2, \cdots, x_n$ are positive integers for which $x_1 + x_2 + \cdots + x_n = 2(n+1)$. Show that there exists an index r with $0 \leq r \leq n-1$ for which the following $n-1$ inequalities hold:

$$x_{r+1} \leq 3$$

$$x_{r+1} + x_{r+2} \leq 5$$

$$\cdots$$

$$x_{r+1} + x_{r+2} + \cdots + r_{r+i} \leq 2i + 1$$

$$\cdots$$

$$x_{r+1} + x_{r+2} + \cdots + x_n \leq 2(n-r) + 1$$

$$\cdots$$

$$x_{r+1} + \cdots + x_n + x_1 + \cdots + x_j \leq 2(n+j-r) + 1$$

$$\cdots$$

$$x_{r+1} + x_{r+2} + \cdots + x_n + x_1 + \cdots + x_{r-1} \leq 2n - 1$$

where $1 \leq i \leq n-r$ and $1 \leq j \leq r-1$. Prove that, if all the inequalities are strict, then r is unique, and that, otherwise, there are exactly two such r.

391. Show that there are infinitely many nonsimilar ways that a square with integer side lengths can be partitioned into three nonoverlapping polygons with integer side lengths which are similar, but no two of which are congruent.

392. Determine necessary and sufficient conditions on the real parameter a, b, c that

$$\frac{b}{cx+a} + \frac{c}{ax+b} + \frac{a}{bx+c} = 0$$

has exactly one real solution.

393. Determine three positive rational numbers x, y, z whose sum s is rational and for which $x - s^3$, $y - s^3$, $z - s^3$ are all cubes of rational numbers.

394. The average age of the students in Ms. Ruler's class is 17.3 years, while the average age of the boys is 17.5 years. Give a cogent argument to prove that the average age of the girls cannot also exceed 17.3 years.