PROBLEMS FOR NOVEMBER

Please send your solution to
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no later than December 15, 2003. It is important that your complete mailing address and your email address appear on the front page.

269. Prove that the number

$$N = 2 \times 4 \times 6 \times \cdots \times 2000 \times 2002 + 1 \times 3 \times 5 \times \cdots \times 1999 \times 2001$$

is divisible by 2003.

270. A straight line cuts an acute triangle into two parts (not necessarily triangles). In the same way, two other lines cut each of these two parts into two parts. These steps repeat until all the parts are triangles. Is it possible for all the resulting triangle to be obtuse? (Provide reasoning to support your answer.)

271. Let x, y, z be natural numbers, such that the number

$$\frac{x - y\sqrt{2003}}{y - z\sqrt{2003}}$$

is rational. Prove that
(a) $xz = y^2$;
(b) when $y \neq 1$, the numbers $x^2 + y^2 + z^2$ and $x^2 + 4z^2$ are composite.

272. Let $ABCD$ be a parallelogram whose area is 2003 sq. cm. Several points are chosen on the sides of the parallelogram.

(a) If there are 1000 points in addition to A, B, C, D, prove that there always exist three points among these 1004 points that are vertices of a triangle whose area is less that 2 sq. cm.

(b) If there are 2000 points in addition to A, B, C, D, is it true that there always exist three points among these 2004 points that are vertices of a triangle whose area is less than 1 sq. cm?

273. Solve the logarithmic inequality

$$\log_4(9^x - 3^x - 1) \geq \log_2 \sqrt{5} \ .$$

274. The inscribed circle of an isosceles triangle ABC is tangent to the side AB at the point T and bisects the segment CT. If $CT = 6\sqrt{2}$, find the sides of the triangle.

275. Find all solutions of the trigonometric equation

$$\sin x - \sin 3x + \sin 5x = \cos x - \cos 3x + \cos 5x \ .$$