PROBLEMS FOR OCTOBER

Please send your solution to
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It is very important that the front page contain your complete mailing address and your email address. The deadline for this set is **November 15, 2002**.

Notes. A function $f : A \rightarrow B$ is a *bijection* iff it is one-one and onto; this means that, if $f(u) = f(v)$, then $u = v$, and, if w is some element of B, then A contains an element t for which $f(t) = w$. Such a function has an *inverse* f^{-1} which is determined by the condition

$$f^{-1}(b) = a \Leftrightarrow b = f(a) \; .$$

178. Suppose that n is a positive integer and that $x_1, x_2, \cdots, x_n$ are positive real numbers such that $x_1 + x_2 + \cdots + x_n = n$. Prove that

$$\sum_{i=1}^{n} \sqrt[n]{ax_i + b} \le a + b + n - 1$$

for every pair a, b or real numbers with all $ax_i + b$ nonnegative. Describe the situation when equality occurs.

179. Determine the units digit of the numbers a^2, b^2 and ab (in base 10 numeration), where

$$a = 2^{2002} + 3^{2002} + 4^{2002} + 5^{2002}$$

and

$$b = 3^1 + 3^2 + 3^3 + \cdots + 3^{2002} \; .$$

180. Consider the function f that takes the set of complex numbers into itself defined by $f(z) = 3z + |z|$. Prove that f is a bijection and find its inverse.

181. Consider a regular polygon with n sides, each of length a, and an interior point located at distances a_1, a_2, $\cdots$, a_n from the sides. Prove that

$$a \sum_{i=1}^{n} \frac{1}{a_i} > 2\pi \; .$$

182. Let ABC be an equilateral triangle with each side of unit length. Let M be an interior point in the equilateral triangle ABC with each side of unit length. Prove that

$$MA.MB + MB.MC + MC.MA \ge 1 \; .$$

183. Simplify the expression

$$\frac{\sqrt{1 + \sqrt{1 - x^2}}((1 + x)\sqrt{1 + x} - (1 - x)\sqrt{1 - x})}{x(2 + \sqrt{1 - x^2})} \; ,$$

where $0 < |x| < 1$.

184. Using complex numbers, or otherwise, evaluate

$$\sin 10^\circ \sin 50^\circ \sin 70^\circ \; .$$