PROBLEMS FOR APRIL

The first five problems appeared on the annual University of Toronto Undergraduate Competition. Please send your solutions to

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no later than **May 15, 2002**.

139. Let A, B, C be three pairwise orthogonal faces of a tetrahedran meeting at one of its vertices and having respective areas a, b, c. Let the face D opposite this vertex have area d. Prove that

$$d^2 = a^2 + b^2 + c^2 \ .$$

140. Angus likes to go to the movies. On Monday, standing in line, he noted that the fraction x of the line was in front of him, while $1/n$ of the line was behind him. On Tuesday, the same fraction x of the line was in front of him, while $1/(n+1)$ of the line was behind him. On Wednesday, the same fraction x of the line was in front of him, while $1/(n+2)$ of the line was behind him. Determine a value of n for which this is possible.

141. In how many ways can the rational 2002/2001 be written as the product of two rationals of the form $(n+1)/n$, where n is a positive integer?

142. Let $x, y > 0$ be such that $x^3 + y^3 \le x - y$. Prove that $x^2 + y^2 \le 1$.

143. A sequence whose entries are 0 and 1 has the property that, if each 0 is replaced by 01 and each 1 by 001, then the sequence remains unchanged. Thus, it starts out as $010010101001 \cdots$. What is the 2002th term of the sequence?

144. Let a, b, c, d be rational numbers for which $bc \neq ad$. Prove that there are infinitely many rational values of x for which $\sqrt{(a+bx)(c+dx)}$ is rational. Explain the situation when $bc = ad$.