International Mathematical Talent Search – Round 33

Problem 1/33. The digits of the three-digit integers a, b, and c are the nine non-zero digits $1, 2, 3, \ldots, 9$, each of them appearing exactly once. Given that the ratio $a : b : c$ is $1 : 3 : 5$, determine a, b, and c.

Problem 2/33. Let $N = 111\ldots1222\ldots2$, where there are 1999 digits of 1 followed by 1999 digits of 2. Express N as the product of four integers, each of them greater than 1.

Problem 3/33. Triangle ABC has angle A measuring $30°$, angle B measuring $60°$, and angle C measuring $90°$. Show four different ways to divide triangle ABC into four triangles, each similar to triangle ABC but with one quarter of the area. Prove that the angles and sizes of the smaller triangles are correct.

Problem 4/33. There are 8436 steel balls, each with radius 1 centimeter, stacked in a tetrahedral pile, with one ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. Determine the height ot the pile in centimeters.

Problem 5/33. In a convex pentagon $ABCDE$ the sides have lengths $1, 2, 3, 4$, and 5, though not necessarily in that order. Let F, G, H, and I be the midpoints of sides AB, BC, CD, and DE, respectively. Let X be the midpoint of segment FH, and Y be the midpoint of segment GI. The length of segment XY is an integer. Find all possible values of the length of side AE.