International Mathematical Talent Search – Round 23

Problem 1/23. In the addition problem on the right, each letter represents a different digit from 0 to 9. Determine them so that the resulting sum is as large as possible. What is the value of GB with the resulting assignment of the digits?

$$
\begin{array}{cccccc}
 & & A & R & L & O \\
 & & B & A & R & T \\
 & & B & R & A & D \\
 & E & L & T & O & N \\
+ & R & O & G & E & R \\
\hline
\end{array}
$$

Problem 2/23. We will say that the integer n is *fortunate* if it can be expressed in the form $3x^2 + 32y^2$, where x and y are integers. Prove that if n is fortunate, then so is $97n$.

Problem 3/23. Exhibit in the plane 19 straight lines so that they intersect one another in exactly 97 points. Assume that it is permissible to have more than two lines intersect at some points. Be sure that your solution should be accompanied by a carefully prepared sketch.

Problem 4/23. Prove that $\cot 10^\circ \cot 30^\circ \cot 50^\circ \cot 70^\circ = 3$.

Problem 5/23. Isosceles triangle ABC has been dissected into thirteen isosceles acute triangles, as shown in the two figures below, where all segments of the same length are marked the same way, and the second figure shows the details of the dissection of $\triangle EFG$. Given that the base angle, θ, of $\triangle ABC$ is an integral number of degrees, determine θ.