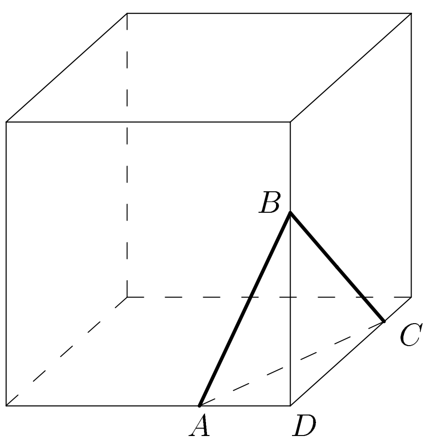
International Mathematical Talent Search – Round 22

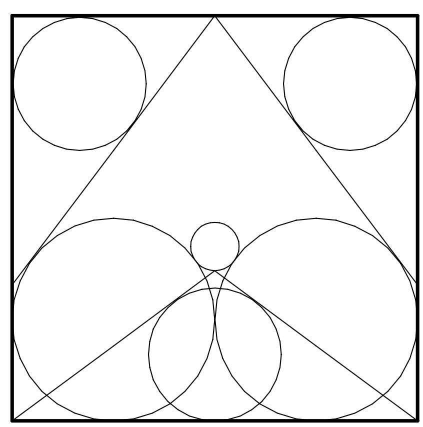
Problem 1/22. In 1996 nobody could claim that on their birthday their age was the sum of the digits of the year in which they were born. What was the last year prior to 1996 which had the same property?

Problem 2/22. Determine the largest positive integer n for which there is a unique positive integer m such that $m < n^2$ and $\sqrt{n+\sqrt{m}}+\sqrt{n-\sqrt{m}}$ is a positive integer.

Problem 3/22. Assume that there are 120 million telephones in current use in the United States. Is it possible to assign distinct 10-digit telephone numbers (with digits ranging from 0 to 9) to them so that any single error in dialing can be detected and corrected? (For example, if one of the assigned numbers is 812–877–2917 and if one mistakenly dials 812–872–2917, then none of the other numbers which differ from 812–872–2917 in a single digit should be an assigned telephone number.)

Problem 4/22. As shown in the first figure below, a large wooden cube has one corner sawed off forming a tetrahedron $ABCD$. Determine the length of CD, if $AD = 6$, $BD = 8$ and area$(\triangle ABC) = 74$.





Problem 5/22. As shown in the second figure above, in a square of base 96 there is one circle of radius r_1, there are two circles of radius r_2, and there are three circles of radius r_3. All circles are tangent to the lines and/or to one another as indicated, and the smallest circle goes through the vertex of the triangle as shown. Determine r_1, r_2, and r_3.