International Mathematical Talent Search – Round 21

Problem 1/21. Determine the missing entries in the magic square shown on the right, so that the sum of the three numbers in each of the three rows, in each of the three columns, and along the two major diagonals is the same constant, k. What is k?

		33
31	28	

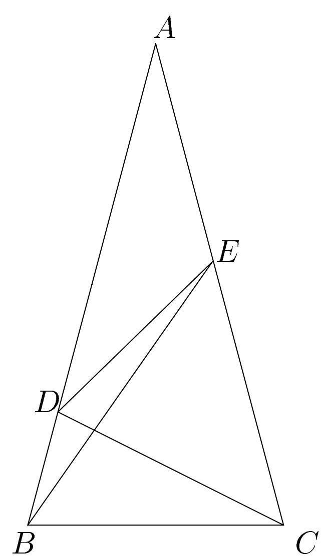
Problem 2/21. Find the smallest positive integer that appears in each of the arithmetic progressions given below, and prove that there are infinitely many positive integers that appear in all three of the sequences.

$$5, 16, 27, 38, 49, 60, 71, \ldots$$
$$7, 20, 33, 46, 59, 72, 85, \ldots$$
$$8, 22, 36, 50, 64, 78, 92, \ldots$$

Problem 3/21. Rearrange the integers $1, 2, 3, 4, \ldots, 96, 97$ into a sequence $a_1, a_2, a_3, a_4, \ldots, a_{96}, a_{97}$, so that the absolute value of the difference of a_{i+1} and a_i is either 7 or 9 for each $i = 1, 2, 3, 4, \ldots, 96$.

Problem 4/21. Assume that the infinite process, shown in the first figure below, yields a well–defined positive real number. Determine this real number.

$$1+\cfrac{1+\cfrac{1+\frac{1+\frac{1+\cdots}{3+\cdots}}{3+\frac{5+\cdots}{1+\cdots}}}{3+\frac{5+\frac{1+\cdots}{3+\cdots}}{1+\frac{5+\cdots}{1+\cdots}}}}{3+\cfrac{5+\frac{1+\frac{1+\cdots}{3+\cdots}}{3+\frac{5+\cdots}{1+\cdots}}}{1+\frac{5+\frac{1+\cdots}{3+\cdots}}{1+\frac{5+\cdots}{1+\cdots}}}}$$



Problem 5/21. Assume that $\triangle ABC$, shown in the second figure above, is isosceles, with $\angle ABC = \angle ACB = 78^\circ$. Let D and E be points on sides AB and AC, respectively, so that $\angle BCD = 24^\circ$ and $\angle CBE = 51^\circ$. Determine, with proof, $\angle BED$.